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RESEARCH OF THE OPTIMAL CONTROL PROBLEM FOR ONE MATHEMATICAL MODEL OF THE SOBOLEV TYPE

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The article is devoted to the study of optimal control for one mathematical model of the Sobolev type, which is based on the model equation, which describes various processes (for example, deformation processes, processes occurring in semiconductors, wave processes, etc.) depending on the parameters and can belong either to the class of degenerate (for $\lambda > 0$) equations or to the class of nondegenerate (for $\lambda < 0$) equations. The article is the first attempt to study the control problem for mathematical semilinear models of the Sobolev type in the absence of the property of non-negative definiteness of the operator at the time derivative, i.e. the construction of a singular optimality system in accordance with the singular situation caused by the instability of the model. Conditions for the existence of a control-state pair are presented, and conditions for the existence of an optimal control are found.

Keywords: Sobolev type equations; phase space method; optimal control problem.

Introduction

Currently, most of modern research is developing at the junction of several areas of knowledge, various processes in engineering, economic, physical phenomena are investigated using methods of mathematical modelling. Briefly characterizing modelling, we note that modelling consists in replacing a real system (process, phenomenon) with a model that is in some correspondence with it (with them) and is able to reproduce the properties or characteristics of a real system. Therefore, the role of mathematical modelling in science, in the research of engineering, economic objects and systems is very large. Note also that carrying out production experiments requires a lot of financial, time and labor costs. Conducting full-scale experiments is sometimes impossible due to a number of reasons, for example, there is no way to control individual parameters (temperature, pressure, the course of processes, or other factors). Therefore, the creation and study of mathematical models describing these processes is of great applied importance.

In most cases, for various physical processes, it is important not only to implement modelling, but also to control the components of the system, in which this process takes place. It is assumed that any dynamical system (i.e. a system that develops in a certain way, evolves in time) at each moment of time can be in a state that belongs to a certain (finite or infinite) set of possible states. In this case, control is understood as an impact that can change the current state, as well as the subsequent development of the system. This raises the question of finding the best control to the process. Consider the optimal control problem

$$J(x,u) \to \inf \tag{1}$$

for a mathematical model of Sobolev type, which is based on the Showalter–Sidorov–Dirichlet problem

$$(\lambda \mathbb{I} + \Delta)(x(s,0) - x_0(s)) = 0, \ s \in \Omega, \tag{2}$$

$$x(s,t) = 0,\ (s,t) \in \partial\Omega \times \mathbb{R}, \tag{3}$$

for the partial differential equation

$$\frac{\partial}{\partial t}(\lambda \mathbb{I} + \Delta)x + \alpha \Delta x + \beta |x|^{p-2} x = u, \quad p \geq 2, \tag{4}$$

where the function $x = x(s,t)$ is the state of the system; $\alpha, \beta,\ \lambda \in \mathbb{R}$ are the parameters of the system, the free term $u = u(s,t)$ corresponds to the external impact.

The derivation and physical description of equation (4) are presented in the work [1] by M.O. Korpusov and A.G. Sveshnikov. Note also that equation (4) is a model equation, which describes various processes (for example, deformation processes, processes occurring in semiconductors, wave processes, etc.) depending on the parameters and can belong either to the class of degenerate (for $\lambda > 0$) equations or to the class of nondegenerate (for $\lambda < 0$) equations. The system under consideration belongs to singular distributed systems, i.e. includes "features" such as instability, break phenomenon, or multiple solutions.

The work [1] considered the case of destruction of the solution to the initial-boundary value problem for equation (1) in a finite time. In other words, the moment of time T_0 depending on the parameters of the problem was found, up to which a solution to the problem under consideration belongs to some functional class, and for $t \geq T_0$ the solution does not belong to the class anymore. Note that equation (4) belongs to the class of Sobolev type semilinear equations, i.e. depending on the parameters of the equation, the operator for the time derivative can degenerate. One of the main methods for studying equations of this type is the phase space method, which was first proposed by G.A. Sviridyuk in [2]. This method allows to investigate the structure of the phase manifold and to identify the domains of existence of the solution. This method consists in reducing the original equation to the nondegenerate equation

$$\dot{x} = F(x)$$

defined on some set of the original space (or in the whole space), which is understood as the phase space of the original equation (phase manifold). Therefore, the study of initial-boundary value problems for various linear and semilinear Sobolev type equations is primarily reduced to the study of their phase spaces. The phase space method was applied in studies of various semi-linear models of mathematical physics [3–5].

As classical works in the theory of optimal control, we note works written by J.-L. Lions. For example, the work [6] systematically studies optimal control problems for partial differential equations. Optimal control problems for linear [7] and nonlinear [8] Sobolev type equations were widely studied. Namely, the optimal control problems for linear equations of Sobolev type were first studied by G.A. Sviridyuk and A.A. Efremov [7]. Currently, various statements of linear control problems were considered and various initial conditions for them were studied. An abstract theory was constructed to study the problems of optimal control to solutions to the Showalter–Sidorov problem with s-monotone and p-coercive nonlinear operator and a nonnegative definite operator for the time derivative [8, 9]. In [10] the necessary and sufficient conditions for the existence and uniqueness of the solution of optimal control problems for high-order Sobolev type equations with an initial-final condition were obtained. In [11] proposed to use methods of the theory of optimal control for solutions problem of recovering adynamically distorted signal. The review [12] is devoted to description mathematical model of the optimal dynamic measurement.

This work is the first attempt to study the control problem for mathematical models of the Sobolev type in the absence of the property of non-negative definiteness of the operator for the time derivative, i.e. following the terminology proposed by J.-L. Lions [13], to construct a singular optimality system in accordance with the singular situation caused by the instability of the model. The article is organised as follows. In Section 1, conditions for the existence of a control-state pair are found. Then, in Section 2, conditions for the existence of an optimal control are constructed.

1. Investigation of Question of Non-Emptiness of Set of Admissible Pairs of Control Problem

Before proceeding to the study of the control problem, we find the conditions under which there exists a set of pairs (x, u) satisfying problem (2) – (4). In order to use the existing approaches to solving problem (2) – (4), we represent equation (4) in the form

$$-\frac{\partial}{\partial t}(\lambda\mathbb{I} + \Delta)x - \alpha\Delta x = \beta|x|^{p-2}x + u, \; p \geq 2, \tag{5}$$

and reduce problem (2) – (4) to the abstract semilinear equation

$$\frac{d}{dt}Lx + Mx = N(x) + u. \tag{6}$$

Consider the functional spaces $\mathfrak{H} = \overset{\circ}{W}{}^1_2(\Omega)$, $\mathfrak{B} = L_p(\Omega)$, $\mathcal{H} = L_2(\Omega)$. Let the space $\mathfrak{H}^* = W_2^{-1}(\Omega)$, $\mathfrak{B}^* = L_q(\Omega)$, $\frac{1}{p} + \frac{1}{q} = 1$. For $n > 2$ and $2 < p \leq 2 + \frac{4}{n-2}$ or for $n = 2$, there are dense and continuous embeddings of the spaces

$$\mathfrak{H} \hookrightarrow \mathfrak{B} \hookrightarrow \mathcal{H} \hookrightarrow \mathfrak{B}^* \hookrightarrow \mathfrak{H}^*. \tag{7}$$

Note that, for $n > 2$ and $2 < p < 2 + \frac{4}{n-2}$ or for $n = 2$, the embedding of the spaces

$$\mathfrak{H} \hookrightarrow \mathfrak{B} \tag{8}$$

is compact. Define the operators L, M and N by the formulas:

$$\langle Lx, y\rangle = \int_\Omega (-\lambda xy + \nabla x \cdot \nabla y)\; ds \;\forall\; x, y \in \mathfrak{H},$$

$$\langle Mx, y\rangle = \alpha \int_\Omega \nabla x \cdot \nabla y \; ds \;\forall\; x, y \in \mathfrak{H},$$

$$\langle N(x), y\rangle = \beta \int_\Omega |x|^{p-2}xy \; ds \;\forall\; x, y \in \mathfrak{B},$$

where $\langle \cdot, \cdot \rangle$ is the scalar product in $\mathcal{H}$. Therefore, taking into account the choice of functional spaces and the construction of operators, problem (2) – (4) is reduced to the Showalter–Sidorov problem

$$L(x(0) - x_0) = 0 \tag{9}$$

for abstract semilinear Sobolev type equation (6).

Denote by $\{\varphi_k\}$ a sequence of eigenfunctions of the homogeneous Dirichlet problem for the Laplace operator $(-\Delta)$ in the domain Ω, and denote by $\{\lambda_k\}$ the corresponding sequence of eigenvalues numbered in non-decreasing order, taking into account the multiplicity. The operators constructed are already well studied. Let us present their main properties.

Lemma 1. (i) *For all $\lambda \in \mathbb{R}$, the operator $L \in \mathcal{L}(\mathfrak{H}, \mathfrak{H}^*)$ is self-adjoint and Fredholm.*
(ii) *For all $\lambda \in (-\infty, \lambda_1]$, the operator $L \in \mathcal{L}(\mathfrak{H}, \mathfrak{H}^*)$ is nonnegative defined.*
(iii) *For all $\alpha \in \mathbb{R}$, the operator $M \in \mathcal{L}(\mathfrak{H}; \mathfrak{H}^*)$ is symmetric and 2-coercive.*
(iv) *For all $\beta \in \mathbb{R}_+$, the operator $N \in C^\infty(\mathfrak{B}; \mathfrak{B}^*)$ is s-monotone and p-coercive.*

Since the operator L is self-adjoint and Fredholm, we identify $\mathfrak{H} \supset \ker L \equiv \operatorname{coker} L \subset \mathfrak{H}^*$. At that,

$$\ker L = \begin{cases} \{0\}, \text{ if } \lambda \neq \lambda_k; \\ \operatorname{span}\{\varphi_k\}, \text{ if } \lambda = \lambda_k, \end{cases}$$

$$\operatorname{im} L = \begin{cases} \mathfrak{H}^*, \text{ if } \lambda \neq \lambda_k; \\ \{x \in \mathfrak{H}^* : \langle x, \varphi_k \rangle = 0\}, \text{ if } \lambda = \lambda_k, \end{cases}$$

$$\operatorname{coim} L = \begin{cases} \mathfrak{H}, \text{ if } \lambda \neq \lambda_k; \\ \{x \in \mathfrak{H} : \langle x, \varphi_k \rangle = 0\}, \text{ if } \lambda = \lambda_k. \end{cases}$$

Hence $\mathfrak{H} = \ker L \oplus \operatorname{coim} L$, $\mathfrak{H}^* = \operatorname{coker} L \oplus \operatorname{im} L$. Further, denote $\ker L = \mathfrak{H}^0$, $\operatorname{coim} L = \mathfrak{H}^1$. Due to the existence of the resulting splitting, we construct the projectors

$$Q = \begin{cases} \mathbb{I}, \text{ if } \lambda \neq \lambda_k; \\ \mathbb{I} - \sum\limits_{\lambda=\lambda_k} \langle \cdot, \varphi_k \rangle, \text{ if } \lambda = \lambda_k. \end{cases}$$

Since problem (2) – (4) belongs to the class of Sobolev type problems, then, in order to study the question of solvability, we use the phase space method proposed by G.A. Sviridyuk [2]. To this end, we construct the set (hereinafter called the phase manifold)

$$\mathfrak{M} = \begin{cases} \mathfrak{H}, \text{ if } \lambda \neq \lambda_k; \\ \{x \in \mathfrak{H} : \alpha \sum\limits_{i=1}^{n} \int\limits_{\Omega} x_{s_i} \varphi_{k s_i}\, ds = \int\limits_{\Omega} (\beta |x|^{p-2} x + u) \varphi_k\, ds\}, \text{ if } \lambda = \lambda_k, \end{cases}$$

and assume

$$(\mathbb{I} - Q)u \text{ does not depend on } t \in (0, T). \tag{10}$$

Definition 1. *A vector-function $x \in C^1([0,T]; \mathfrak{H})$ is said to be*
– a classical solution to equation (6), if x satisfies the equation for all $t \in (0, T)$;
– a weak generalized solution to equation (6), if, for some $T \in \mathbb{R}_+$, the vector-function x satisfies

$$\int\limits_0^T < \frac{d}{dt} Lx + Mx, y > dt = \int\limits_0^T < N(x) + u, y > dt \ \forall y \in L_2(0, T; \mathfrak{H}).$$

A solution $x = x(t)$ to equation (6) is called a solution to the Showalter–Sidorov problem if x also satisfies the initial condition (9).

In the works of G.A. Sviridyuk and A.A. Efremov [7], it was proposed to consider the space $H^1(\mathfrak{H}) = \{x \in L_2(0,T;\mathfrak{H}) : \dot{x} \in L_2(0,T;\mathfrak{H})\}$ and it was noted that, due to the continuity of the embedding $H^1(\mathfrak{H}) \hookrightarrow C([0,T];\mathfrak{H})$, if there exists a classical solution to problem (6), (9), then the solution is both a strong solution to this problem, and, in our case, a weak generalized solution.

Remark 1. In the works of G.A. Sviridyuk and T.G. Sukacheva [14], the concept of a quasistationary (semi) trajectory was first introduced for solving the Cauchy problem for a semilinear Sobolev type equation. In our work, the solution $x = x(t)$ to equation (6) is called a *quasistationary semitrajectory of the equation*, if $x(t) \in \mathfrak{M}$ for all $t \in (0,T]$, where the phase manifold $\mathfrak{M} = \{x \in \mathfrak{H} : (\mathbb{I} - Q)M = (\mathbb{I} - Q)N + (\mathbb{I} - Q)u\}$, which is a special case of the concept introduced earlier. If a quasi-stationary semi-trajectory satisfies condition (9), then the semi-trajectory is called a quasi-stationary semitrajectory of equation (6) passing through the point x_0.

To investigate the solvability of problem (2) – (4), following the phase space method, we show the simplicity of the phase manifold. From the simplicity of the phase manifold, based on the classical theorem of existence and uniqueness of a solution to the Cauchy problem for a non-degenerate equation [15], we obtain the conditions for the existence of a classical local solution to the problem under study.

Definition 2. *Fix $x_0 \in \mathfrak{M}$, define $x_0^1 = Px_0$, then $x_0^1 \in \mathfrak{H}^1$. The set $\mathfrak{M}$ is a Banach C^k-manifold at the point x_0, if there exist neighborhoods $\mathfrak{O}_0^{\mathfrak{M}} \subset \mathfrak{M}$ and $\mathfrak{O}_0^1 \subset \mathfrak{H}^1$ of the points x_0 and x_0^1, respectively, and C^k-diffeomorphism $\delta : \mathfrak{O}_0^1 \to \mathfrak{O}_0^{\mathfrak{M}}$ such that δ^{-1} is equal to the restriction of the projector P to $\mathfrak{O}_0^{\mathfrak{M}}$. The set $\mathfrak{M}$ is called a Banach C^k-manifold modeled by the subspace $\mathfrak{H}^1$, if $\mathfrak{M}$ is a Banach C^k-manifold at every point. A connected Banach C^k-manifold is called simple if any of its atlas is equivalent to an atlas, containing a single map.*

Based on abstract results, the following statement is true.

Theorem 1. [3] *If, for $\lambda \in \mathbb{R}$, condition* (10) *is satisfied and $\alpha \in \mathbb{R}$, $\beta \in \mathbb{R}$, moreover $\alpha\beta < 0$, and $n > 2$, $2 < p \leq 2 + \frac{4}{n-2}$ or $n = 2$, then*

(i) the set $\mathfrak{M}$ is a simple Banach C^1-manifold modeled by the subspace $\mathfrak{H}^1$;

(ii) there exist $T(x_0)$ and a unique quasistationary semitrajectory $x \in C^1(0,T(x_0);\mathfrak{M})$ of equation (4) *passing through the point x_0.*

Theorem 1 formulates conditions for the existence of a classical local solution to problem (2) – (4). In the works of G.A. Sviridyuk [5,16], conditions were found under which a nonlocal solution to the problem exists. But in this case, additional conditions of positive definiteness are imposed on the operator L. This condition, together with the conditions of the s-monotonicity and p-coercivity of the operator N, allow to obtain conditions under which a solution is extended in time. Note the importance of the requirement that each term on the right-hand side of the equation is positive definite, that is,

$$< Lx, x > \geq 0 \text{ and } < Mx, x >> 0 \text{ and } < N(x), x >> 0 \, \forall x \in \mathfrak{H}.$$

If this condition is not satisfied, then the phenomena of destruction of the solution are observed. Let us impose additional conditions on the parameter λ and obtain conditions under which a solution is extended in time. Consider the following theorem, the proof of which is similar [16].

Theorem 2. *Suppose that condition* (10) *holds for* $\lambda \in (-\infty, \lambda_1]$, *the parameters are* $\alpha \in \mathbb{R}_+$, $\beta \in \mathbb{R}_-$ *and* $n > 2$, $2 \le p \le 2 + \frac{4}{n-2}$ *or* $n = 2$. *Then, for any* $T > 0$, *there exists a unique quasistationary semitrajectory* $x \in C^1((0,T); \mathfrak{M})$ *of equation* (4) *passing through the point* x_0.

2. Control Problem

In the cylinder $Q_T = \Omega \times (0,T)$, consider optimal control problem (1) – (4) in the case of specifying the objective functional in the form

$$J(x,u) = \varkappa \int_0^T \|x - z_d\|^p_{\mathfrak{B}} dt + (1-\varkappa) \int_0^T \|u\|^q_{\mathfrak{B}^*} dt. \tag{11}$$

Introduce a space of controls $\mathfrak{U} = L_q(0,T;\mathfrak{B}^*)$ and choose a non-empty, closed, convex set $\mathfrak{U}_{ad}$. Also, note that by a solution to problem (1) – (4) we mean a pair of functions $(\hat{x},\ \hat{u})$ that satisfies the following condition:

$$J(\hat{x},\hat{u}) = \inf_{(x,u)} J(x,u),$$

where the pairs $(\hat{x},\hat{u}) \in \mathfrak{X} \times \mathfrak{U}_{ad}$ satisfy problem (2) – (4).

Remark 2. Let $\mathfrak{X} = \{x \in L_2(0,T;\mathfrak{H}) : \dot{x} \in L_2(0,T;\operatorname{coim} L)\}$. Denote by $\mathfrak{A}$ the set of pairs $(x,u) \subset \mathfrak{X} \times \mathfrak{U}_{ad}$ satisfying problem (2) – (4). The non-emptiness of the set $\mathfrak{A}$ is guaranteed by the results of the previous section. If $\mathfrak{U}_{ad} = \varnothing$, then, for any $u \in \mathfrak{U}_{ad} \subset \mathfrak{U}$, the set of admissible pairs (x,u) is not empty.

Theorem 3. *Suppose that* $\lambda \in \mathbb{R}$, *the parameters are* $\alpha \in \mathbb{R}$, $\beta \in \mathbb{R}$, *moreover* $\alpha\beta < 0$, *and* $n > 2$, $2 \le p \le 2 + \frac{4}{n-2}$ *or* $n = 2$, *and condition* (10) *holds. Then, for any* $x_0 \in \mathfrak{H}$, $T \in \mathbb{R}_+$ *such that the set* $\mathfrak{A} \neq \emptyset$, *there exists a solution to optimal control problem* (1) – (4).

Proof. Since the set $\mathfrak{A} \neq \emptyset$ is nonempty, there exists a pair $(\tilde{x},\tilde{u}) \in \mathfrak{X} \times \mathfrak{U}_{ad}$ such that $\{\tilde{x},\tilde{u}\}$ satisfy the problem. Then

$$\inf_{\{x,u\}} J(x,u) \le J(\tilde{x},\tilde{u})$$

there exists a minimizing sequence $\{x_m, u_m\} \subset \mathfrak{X} \times \mathfrak{U}_{ad}$ such that

$$J(x_m,u_m) \le \text{const}.$$

Since functional (11) has the coercivity property, then

$$\|x_m\|_{L_p(0,T;\mathfrak{B})} \le \text{const}, \quad \|u_m\|_{L_q(0,T;\mathfrak{B}^*)} \le \text{const} \tag{12}$$

for all $m \in \mathbb{N}$. By virtue of (12) (passing, if necessary, to subsequences), we extract sequences, which are weakly converging in the corresponding spaces:

$$x^m \rightharpoonup \hat{x} \text{ is weak in } L_p(0,T;\mathfrak{B});$$

$$u^m \rightharpoonup \hat{u} \text{ is weak in } L_q(0,T;\mathfrak{B}^*).$$

Due to the p-coercivity of the operator N, we obtain

$$\int_0^T \langle N(x_m), x_m\rangle \, d\tau \leq \int_0^T \|N(x_m)\|_{\mathfrak{B}^*}\|x_m\|_{\mathfrak{B}} d\tau \leq C\int_0^T \|x_m\|_{\mathfrak{B}}^{p-1}\|x_m\|_{\mathfrak{B}},$$

and, therefore, $N(x_m)$ are bounded in $L_q(0,T;\mathfrak{B}^*)$. Then, we use (4) to obtain that

$$-\frac{\partial}{\partial t}(\lambda\mathbb{I} + \Delta)x_m - \alpha\Delta x_m = \beta|x_m|^{p-2}x_m + u_m, \tag{13}$$

and, taking into account that the right-hand side of (13) is bounded in $L_q(0,T;\mathfrak{B}^*)$, we arrive at the boundedness

$$\|x_m\|_{\mathfrak{X}} \leq \text{const}.$$

We extract the weakly convergent subsequence

$$x^m \rightharpoonup \hat{x} \text{ is weak in } \mathfrak{X}.$$

By Mazur's theorem and the sequential weak closedness of the set $\mathfrak{U}_{ad}$, the point $\hat{u} \in \mathfrak{U}_{ad}$. Let us pass to the limit in equation of state (4) and obtain

$$[L\frac{d\hat{x}}{dt} + M\hat{x}, \zeta] = [\mu + \hat{u}, \zeta]. \tag{14}$$

By virtue of reflexivity of the space $L_q(0,T;\mathfrak{B}^*)$ and compactness of embeddings (Aubin – Lions lemma)

$$W = \{x \in L_2(0,T;\mathfrak{H}) : \dot{x} \in L_2(0,T;\mathcal{H})\} \hookrightarrow L_2(0,T;\mathfrak{B}),$$

we obtain that

$$N(x) = \mu.$$

Therefore, $\hat{x} = \hat{x}(\hat{u})$ and $\liminf J(x_m, u_m) \geq J(\hat{x}, \hat{u})$. Hence, $\hat{u}$ is the optimal control in problem (2) – (4). □

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ИССЛЕДОВАНИЕ ЗАДАЧИ ОПТИМАЛЬНОГО УПРАВЛЕНИЯ РЕШЕНИЯМИ ОДНОЙ МАТЕМАТИЧЕСКОЙ МОДЕЛИ

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Статья посвящена исследованию оптимального управления для одной математической модели соболевского типа, базирующаяся на модельном уравнении, которое описывает различные процессы (например, процессы деформации, процессы, происходящие в полупроводниках, волновые процессы и т. д.) в зависимости от параметров и может принадлежать либо к классу вырожденных (при $\lambda > 0$) уравнений, либо к классу невырожденных (для $\lambda < 0$) уравнений. Статья является первой попыткой исследования задачи управления для математических полулинейных моделей соболевского типа в случае отсутствия свойства неотрицательной определенности оператора при производной по времени, т.е. построению сингулярной системы оптимальности в соответствии с сингулярной ситуацией, обусловленной неустойчивостью модели. Представлены условия существования пары управления-состояния, а также найдены условия существования оптимального управления.

Ключевые слова: уравнения соболевского типа; метод фазового пространства; задача оптимального управления.

Литература

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