The article discusses a mathematical model of the production activity of PJSC “Fortum”, which is the main supplier of electrical and thermal energy in the Southern Urals. The model is constructed on the basis of the modified production function of Cobb-Douglas for 2010-2020. The author’s algorithm for constructing the model is given, according to which a computer program for modelling the operating activities of an enterprise was written and registered in the State Register of computer programs in the Java language. The adequacy of the constructed model was checked by the coefficient of determination, the value of which showed its high reliability. The analysis of the constructed mathematical model made it possible to draw the following conclusions: the strategic goal of reducing tariffs declared in the course of the restructuring of OJSC “RAO UES of Russia” was not fulfilled, on the contrary, their steady growth is observed; the efficiency of operating activities does not meet modern requirements – during the entire analyzed period, there is a diminishing economies of scale; the sustainable development strategy implemented by PJSC “Fortum” does not have a proper economic power supply unit due to a decrease in the return on resources involved in production; relatively high financial results of PJSC “Fortum” are associated with an unreasonable increase in tariffs; consumers of electric and thermal energy of the Urals and Western Siberia, in the conditions of the monopoly on the energy market, finance the richest country in the world – the Netherlands – through payments from Fortum Holding B.V., which directly and indirectly owns 98% of the shares of PJSC “Fortum”; with the existing management mechanism and state control by the antimonopoly service, PJSC “Fortum” has no incentives to replace the outdated cogeneration option based on hydrocarbon fuels with modern, progressive green technologies. The results of the study are recommended to the Federal Antimonopoly Service to strengthen control over the activities of PJSC “Fortum”, reduce tariff pressure on consumers of electric and thermal energy and unjustified enrichment of foreign subjects of the electric power industry at the expense of Russian consumers.

**Keywords:** model; analysis; energy market; energy supply efficiency; state control.

**Introduction**

A little more than 20 years passed since the beginning of the reform of the electric power industry in Russia. During this time, the relevance of research on energy markets was not weakened [1–3]. In this work, we made an attempt to assess the activities of the main supplier of electrical and thermal energy in the Southern Urals.

As part of the restructuring of OJSC “RAO UES of Russia”, OJSC “TGK-10” was reorganized into OJSC (subsequent PJSC) Fortum. Production assets and activities are concentrated in the Urals and Western Siberia. Since 2010, PJSC “Fortum” implemented
an extensive investment program in Russia and constructed 11 modern and highly efficient power units with a total capacity of 2,4 GW, 8 of which are part of the current program for the modernization of Russia’s energy capacities. Over 10 years of operation, the capacity of the company’s energy facilities was almost doubled. At the end of 2020, the net profit of Fortum PJSC amounted to RUB 16.6 billion. But are the results of operating and financial activities of the company in the Russian energy market absolutely so good? Let us answer this question, for which we construct a mathematical model of the operational activities of PJSC “Fortum”.

1. Algorithm for Modelling

To construct a mathematical model of the enterprise’s activities based on the modified Cobb-Douglas production function, taking into account the autonomous technical progress, neutral according to Hicks (1), the author’s algorithm is used, as described in [4]:

$$\text{CP} = A \cdot N^\alpha \cdot F A^\beta \cdot C A^\gamma \cdot e^{\lambda t},$$

(1)

where $e^{\lambda t}$ is a factor that allows to take into account the “autonomous” technical progress, neutral according to Hicks (here $e$ is the base of the natural logarithm).

The production function (1) is an economic and mathematical model of production reflecting the impact of resource provision of production on output. In the most general form, the parameters of the production function $A, \alpha, \beta, \gamma, \lambda$ are found on the basis of retrospective data on revenue CP, the number of employees N, non-current assets FA, current assets CA, and the corresponding time $t$, as a solution to the system of equations (2). In the system of equations (2), $m$ is the number of years for which retrospective data were collected ($m > 4$).

\[
\begin{aligned}
\sum_{i=1}^{m} \ln CP_i &= m \cdot \ln A + \alpha \cdot \sum_{i=1}^{m} \ln N_i + \beta \cdot \sum_{i=1}^{m} FA_i + \gamma \cdot \sum_{i=1}^{m} \ln CA_i + \\
&+ \lambda \cdot \sum_{i=1}^{m} t_i, \\
\sum_{i=1}^{m} (\ln CP_i \cdot \ln N_i) &= \ln A \cdot \sum_{i=1}^{m} \ln N_i + \alpha \cdot \sum_{i=1}^{m} (\ln N_i)^2 + \\
&+ \beta \cdot \sum_{i=1}^{m} (\ln FA_i \cdot \ln N_i) + \gamma \cdot \sum_{i=1}^{m} (\ln CA_i \cdot \ln N_i) + \lambda \cdot \sum_{i=1}^{m} (t_i \cdot \ln N_i), \\
\sum_{i=1}^{m} (\ln CP_i \cdot \ln FA_i) &= \ln A \cdot \sum_{i=1}^{m} \ln FA_i + \alpha \cdot \sum_{i=1}^{m} (\ln FA_i \cdot \ln N_i) + \\
&+ \beta \cdot \sum_{i=1}^{m} (\ln FA_i)^2 + \gamma \cdot \sum_{i=1}^{m} (\ln FA_i \cdot \ln CA_i) + \lambda \cdot \sum_{i=1}^{m} (t_i \cdot \ln FA_i), \\
\sum_{i=1}^{m} (\ln CP_i \cdot \ln CA_i) &= \ln A \cdot \sum_{i=1}^{m} \ln CA_i + \alpha \cdot \sum_{i=1}^{m} (\ln CA_i \cdot \ln N_i) + \\
&+ \beta \cdot \sum_{i=1}^{m} (\ln CA_i \cdot \ln FA_i) + \gamma \cdot \sum_{i=1}^{m} (\ln CA_i)^2 + \lambda \cdot \sum_{i=1}^{m} (t_i \cdot \ln CA_i), \\
\sum_{i=1}^{m} (\ln CP_i \cdot t_i) &= \ln A \cdot \sum_{i=1}^{m} t_i + \alpha \cdot \sum_{i=1}^{m} (t_i \cdot \ln N_i) + \\
&+ \beta \cdot \sum_{i=1}^{m} (t_i \cdot \ln FA_i) + \gamma \cdot \sum_{i=1}^{m} (t_i \cdot \ln CA_i) + \lambda \cdot \sum_{i=1}^{m} (t_i)^2.
\end{aligned}
\]

(2)

The use of the method of power-law production functions for analyzing the economics of production is often complicated by the fact that system of equations (2) may not have
Let us divide the full differential of function (1) by the function itself. We get:

\[ \frac{dCP}{CP} = \alpha \cdot dN/N + \beta \cdot \frac{dFA}{FA} + \gamma \cdot \frac{dCA}{CA} + \lambda \cdot dt. \]  

(3)

Let us introduce the notation

\[ \frac{dCP}{CP} = 2 \cdot \frac{CP_{i+1} - CP_i}{CP_{i+1} + CP_i} = \frac{dN}{N} = 2 \cdot \frac{N_{i+1} - N_i}{N_{i+1} + N_i} = x_i dt = t_{i+1} - t_i = 1, \]

\[ \frac{dFA}{FA} = 2 \cdot \frac{FA_{i+1} - FA_i}{FA_{i+1} + FA_i} = y, \quad \frac{dCA}{CA} = 2 \cdot \frac{CA_{i+1} - CA_i}{CA_{i+1} + CA_i} = w. \]

Then expression (3) is converted into the equation

\[ z = \alpha \cdot x + \beta \cdot y + \gamma \cdot w + \lambda. \]  

(4)

Based on the transformed initial data from system of equations (5), we find the elasticity coefficients \( \alpha, \beta, \gamma, \lambda \)

\[
\begin{align*}
\sum_{i=1}^{m} z_i &= \lambda \cdot m + \alpha \cdot \sum_{i=1}^{m} x_i + \beta \cdot \sum_{i=1}^{m} y_i + \gamma \cdot \sum_{i=1}^{m} w_i, \\
\sum_{i=1}^{m} (x_i \cdot z_i) &= \lambda \cdot \sum_{i=1}^{m} x_i + \alpha \cdot \sum_{i=1}^{m} x_i^2 + \beta \cdot \sum_{i=1}^{m} (x_i \cdot y_i) + \\
&+ \gamma \cdot \sum_{i=1}^{m} (x_i \cdot w_i), \\
\sum_{i=1}^{m} (y_i \cdot z_i) &= \lambda \cdot \sum_{i=1}^{m} y_i + \alpha \cdot \sum_{i=1}^{m} x_i \cdot y_i + \beta \cdot \sum_{i=1}^{m} (y_i^2) + \\
&+ \gamma \cdot \sum_{i=1}^{m} (y_i \cdot w_i), \\
\sum_{i=1}^{m} (w_i \cdot z_i) &= \lambda \cdot \sum_{i=1}^{m} w_i + \alpha \cdot \sum_{i=1}^{m} w_i \cdot x_i + \beta \cdot \sum_{i=1}^{m} (y_i \cdot w_i) + \\
&+ \gamma \cdot \sum_{i=1}^{m} (w_i^2). \\
\end{align*}

\]

(5)

The found numerical values of the elasticity coefficients are used to find the coefficient

\[ A = \frac{\sum_{i=1}^{m} z_i \cdot x_i^\alpha \cdot y_i^\beta \cdot w_i^\gamma \cdot e^{\lambda t}}{\sum_{i=1}^{m} (x_i^\alpha \cdot y_i^\beta \cdot w_i^\gamma \cdot e^{\lambda t})^2}. \]  

(6)

2. Construction of Mathematical Model of PJSC “Fortum”

The described modelling algorithm was formalized in the form of a computer program written in Java and registered by the Federal Service for Intellectual Property in the State

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a solution. This is due to the fact that there may be a relationship between statistical data, due not so much to their functional relationship, but to the proximity in time of the sets of exogenous variables, when all values change proportionally. This gives rise to a phenomenon called by Mendershausen the effect of multicollinearity between independent variables. To overcome this barrier, it is necessary to make the following transformations.

The found numerical values of the elasticity coefficients are used to find the coefficient

\[ A = \frac{\sum_{i=1}^{m} z_i \cdot x_i^\alpha \cdot y_i^\beta \cdot w_i^\gamma \cdot e^{\lambda t}}{\sum_{i=1}^{m} (x_i^\alpha \cdot y_i^\beta \cdot w_i^\gamma \cdot e^{\lambda t})^2}. \]  

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2. Construction of Mathematical Model of PJSC “Fortum”

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V.G. Mokhov, G.S. Chebotareva

Table

Financial statements of PJSC “Fortum”

<table>
<thead>
<tr>
<th>Year</th>
<th>CP, thousand roubles</th>
<th>FA, thousand roubles</th>
<th>CA, thousand roubles</th>
<th>N, people</th>
</tr>
</thead>
<tbody>
<tr>
<td>2010</td>
<td>27 715 940</td>
<td>13 684 953</td>
<td>26 247 702</td>
<td>2 793</td>
</tr>
<tr>
<td>2011</td>
<td>32 134 830</td>
<td>64 533 203</td>
<td>22 323 791</td>
<td>2 693</td>
</tr>
<tr>
<td>2012</td>
<td>36 841 863</td>
<td>82 901 799</td>
<td>19 389 342</td>
<td>2 781</td>
</tr>
<tr>
<td>2013</td>
<td>44 193 363</td>
<td>97 284 273</td>
<td>17 474 427</td>
<td>2 781</td>
</tr>
<tr>
<td>2014</td>
<td>50 682 405</td>
<td>109 514 102</td>
<td>22 354 404</td>
<td>2 835</td>
</tr>
<tr>
<td>2015</td>
<td>54 076 392</td>
<td>124 196 788</td>
<td>20 687 133</td>
<td>2 890</td>
</tr>
<tr>
<td>2016</td>
<td>60 471 373</td>
<td>130 953 002</td>
<td>27 170 175</td>
<td>3 745</td>
</tr>
<tr>
<td>2017</td>
<td>65 281 414</td>
<td>134 650 018</td>
<td>37 795 189</td>
<td>2 363</td>
</tr>
<tr>
<td>2018</td>
<td>73 807 330</td>
<td>128 778 174</td>
<td>40 147 946</td>
<td>2 290</td>
</tr>
<tr>
<td>2019</td>
<td>76 787 529</td>
<td>125 260 027</td>
<td>30 767 407</td>
<td>2 298</td>
</tr>
<tr>
<td>2020</td>
<td>78 538 723</td>
<td>119 576 134</td>
<td>35 913 046</td>
<td>2 329</td>
</tr>
</tbody>
</table>

Register of Computer Programs. The initial modelling base was the data from a sample of the accounting statements of PJSC “Fortum” for 2010–2020 presented in Table.

As a result of modelling, the following mathematical model of the operational activities of PJSC “Fortum” was obtained:

\[ CP = 920737 \cdot N^{0.11976} \cdot FA^{0.10086} \cdot CA^{0.04533} \cdot e^{0.0919t}. \]  

(7)

The reliability of the constructed model is estimated by the coefficient of determination

\[ R^2 = 1 - \frac{\sum e_i^2}{\sum(y_i - \bar{y})^2}. \]  

(8)

The determination coefficient takes values in the range from 0 to 1, and the closer the coefficient value to 1, the better the constructed model describes the initial data. For constructed model (7), the value of the determination coefficient is 0.91, which corresponds to its high accuracy.

Conclusions

The indicator calculated in the modelling as \( h = \alpha + \beta + \gamma \) is called the elasticity of production. Its value shows how the scale of production affects output. If \( h = 1 \), then function (7) assumes a constant effect of growth in the scale of production. If \( h > 1 \), the growing effect of an increase in the scale of production prevails, i.e. in this production process, the return on production resources increases. If \( h < 1 \), a diminishing effect of an increase in the scale of production is manifested, that is, the return of resources involved in production at the enterprise decreases. In resulting model (7), \( h = 0.26595 \), which indicates a diminishing effect of an increase in the scale of production.

The dynamics of the indicator \( h \) can be used to assess the economic sustainability of an enterprise. If \( \partial h/\partial t > 0 \), for \( t \in (1, T) \), then there is the economic stability of the
enterprise, otherwise we see a decrease in the economic stability of the enterprise. Since 2015, the indicator of the dynamics of the elasticity of production of PJSC “Fortum” is more than 0, which indicates the economic stability of the enterprise. However, the economic return of the resources used is insufficient to finance the social and environmental sectors of sustainable development.

The results obtained allow us to draw the following conclusions: the strategic goal of reducing tariffs declared in the course of the restructuring of OJSC “RAO UES of Russia” was not fulfilled, on the contrary, their steady growth is observed; the efficiency of operating activities does not meet modern requirements – during the entire analyzed period, there is a diminishing economies of scale; the sustainable development strategy implemented by PJSC “Fortum” does not have a proper economic power supply unit due to a decrease in the return on resources involved in production; relatively high financial results of PJSC “Fortum” are associated with an unreasonable increase in tariffs; consumers of electric and thermal energy of the Urals and Western Siberia, in the conditions of the monopoly on the energy market, finance the richest country in the world – the Netherlands – through payments from Fortum Holding B.V., which directly and indirectly owns 98% of the shares of PJSC “Fortum”; with the existing management mechanism and state control by the antimonopoly service, PJSC “Fortum” has no incentives to replace the outdated cogeneration option based on hydrocarbon fuels with modern, progressive green technologies.

References


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В статье рассмотрена математическая модель производственной деятельности основного поставщика электрической и тепловой энергии Южного Урала – ПАО «Фортум», построенная на основе модифицированной производственной функции Кобба – Дугласа за 2009–2020 годы. Приведен авторский алгоритм построения модели, по которому на языке Java написана и зарегистрирована в государственном Реестре программ для ЭВМ компьютерная программа моделирования операционной деятельности предприятия. Адекватность построенной модели проверена по коэффициенту детерминации, значение которого показало ее высокую достоверность. Анализ построенной математической модели позволил сделать следующие выводы: заявленное в ходе реформирования ОАО «РАО ЕЭС России» стратегическая цель по снижению тарифов не выполнена, наоборот наблюдается их неуклонный рост; эффективность операционной деятельности не отвечает современным требованиям – в течение всего анализируемого периода имеет место убывающий эффект масштаба; стратегия устойчивого развития, реализуемая ПАО «Фортум», не имеет должного экономического питающего блока из-за снижения отдачи вовлекаемых в производство ресурсов; относительно высокие финансовые результаты деятельности ПАО «Фортум» связаны с необоснованным ростом тарифов; потребители электрической и тепловой энергии Урала и За- падной Сибири в условиях сложившейся на энергетическом рынке монополии финансуют самую богатую страну в мире – Нидерланды, через платежи компании Fortum Holding B.V., которая владеет напрямую и опосредовано 98 % акций ПАО «Фортум»; при сложившемся механизме хозяйствования и государственном контроле со стороны антимонопольной службы у ПАО «Фортум» отсутствуют стимулы для замены устаревшего варианта когенерации на основе углеводородного топлива современными, прогрессивными зелеными технологиями.

Результаты исследования рекомендуются Федеральной антимонопольной службе для усиления контроля за деятельностью ПАО «Фортум», снижению тарифного давления на потребителей электрической и тепловой энергии и необоснованного обогащения иностранных субъектов электроэнергетики за счет потребителей России.

Ключевые слова: модель; анализ; энергетический рынок; эффективность энергоснабжения; государственный контроль.

Литература


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