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TWO-STAGE PARAMETRIC IDENTIFICATION PROCEDURE FOR A SATELLITE MOTION MODEL BASED ON ADAPTIVE UNSCENTED KALMAN FILTERS

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The paper presents a new two-stage parametric identification procedure for constructing a navigation satellite motion model. At the first stage of the procedure, the parameters of the radiation pressure model are estimated using the maximum likelihood method and the multiple adaptive unscented Kalman filter. At the second stage, the parameters of the unaccounted perturbations model are estimated based on the results of residual differences measurements. The obtained results lead to significant improvement of prediction quality of the satellite trajectory.

Keywords: nonlinear stochastic continuous-discrete system; multiple adaptive unscented Kalman filter; parametric identification; ML method; satellite orbital motion model.

Introduction

Description of a system in terms of nonlinear mathematical models allows to take into account additional factors due to nonlinear laws of nature and to conduct a better analysis of objects. Obtaining a model with good predicting properties requires informative measurement data and a suitable model structure capable of accurately describing the dynamics of the process; therefore, when constructing models of nonlinear systems, parametric identification methods are used. Traditionally, the maximum likelihood (ML) method is used to solve the problem of parametric identification [1–3]. In case of using dynamic models with Gaussian noise, the corresponding identification criterion is written on the basis of the equations of the extended Kalman filter (EKF) [4]. Although the EKF is widely used, this filter has some drawbacks. The filter applies the standard linear Kalman filter technique to linearize a nonlinear model. It requires the sufficient differentiability of the dynamic state and the susceptibility to biasing and to divergence of the state estimates. This approach is sub-optimal and can easily lead to the divergence. These difficulties can be successfully overcome with such nonlinear filters as the cubature Kalman filter [5,6] and the unscented Kalman filter (UKF) [7–10]. S.J. Julier et al. [7] proposed the UKF as a derivative free alternative to the extended Kalman filter in the framework of state estimation. Statistical parameters of noise are set inaccurately or they are completely unknown, when solving practical problems. The presence of outliers in the measurement data makes the further determination of such characteristics complicated. When using the incorrect a priori information about the noise properties of the system and/or the measurements, the obtained estimates may be biased. The covariance matrices of the system and the measurements noises are usually selected accordingly to the results of some empirical data analysis or the various situations modelling. The correct specification of the statistical noise parameters often determines the accuracy of the state vector estimation. One of the possible solutions to this problem is using adaptive methods for the measurement data processing [11–16], which, along with the state vector estimation, can restore the statistical characteristics of noises. Currently, there exists a sufficient number of publications in which adaptive modifications of UKF are given. However, it was shown in [17] that the use of one of the adaptive modifications of UKF in the construction of satellite orbital motion model is not possible. In this regard, the paper proposes to use several adaptive filters together when solving the parametric identification problem. In this research, two different modifications of UKF (adaptive block I [16–19] and adaptive block II [17,20]) are combined with the UKF algorithm to evaluate and improve the statistical properties of process noise. This correction can result in reducing the model error, suppressing the filtering divergence and improving the filtering accuracy. The idea of a two-stage identification procedure was considered in [21]. At the first stage of the procedure, all unknown parameters of the motion and measurements models are evaluated based on the least squares method (LSM). Next, we calculate the difference (discrepancy) between the measured values and those calculated from the obtained model of motion and measurements. At the second stage, linearized motion and measurement models are used for corrections. The motion model includes additional accelerations described by the first-order Gauss–Markov process. The parameters of the Gauss–Markov process, together with corrections to the motion vector of the apparatus, are determined using the Kalman filter. For the difference between the LSM estimates and estimates based on the Kalman filter, we construct a function on the measurement interval, which is then used for predicting. In this paper, new algorithms for two-stage parametric identification are proposed. For a more accurate construction of the mathematical model, it is proposed to additionally evaluate disturbances based on the results of measurements of residual differences by finding estimates of radiation pressure parameters.

1. Motion Model of Navigation Satellite

The quality of the ephemeris-temporal support for Global Navigation Satellite System (GNSS) technologies depends on adequacy of the applied mathematical models describing the orbital motion of navigation satellites. Consider the following stochastic nonlinear continuous-discrete model of orbital motion of the navigation satellite and model measurement [22,23]:

$$\frac{d}{dt}(\dot{r}(t)) = \underbrace{-\frac{\mu \cdot M_E}{\|r(t)\|^3} r(t) + g_1(r(t)) + g_2(r(t)) + g_3(r(t), \dot{r}(t), \theta)}_{f(R(t))} + w(t), \quad t \in [t_0, t_N], \tag{1}$$

$$s(t_{k+1}) = h(R(t_{k+1})) + \nu(t_{k+1}), \quad k = 0, 1, \ldots, N-1, \tag{2}$$

where $R(t) = \begin{pmatrix} r(t) \\ \dot{r}(t) \end{pmatrix}$, $r(t) = ((x(t), y(t), z(t))^T$ is the coordinate vector of the navigation satellite in an inertial coordinate system, $\dot{r}(t) = (V_x(t), V_y(t), V_z(t))^T$ is the velocity vector of the navigation satellite in an inertial coordinate system; $f(R(t)), h(R(t_{k+1}))$ are nonlinear functions, where μ is the gravitational constant, M_E is the mass of the Earth; $\|r(t)\| = \sqrt{x^2(t) + y^2(t) + z^2(t)}$ is the radius of the orbit, $\|\cdot\|$ is the Euclidean vector norm, $g_1(r(t))$ is the perturbations, caused by the non-sphericity of the Earth's geopotential, $g_2(r(t))$ are the perturbations caused by the gravitational influence of the Moon, the Sun and/or the other planets, $g_3(r(t), \dot{r}(t), \theta)$ are perturbations from the solar radiation; $\theta \in \Omega_\theta$ is the vector of unknown parameters; $s(t_{k+1})$ is the measurement vector (for example, pseudorange, query range, satellite laser ranging (SLR) from ground points to the navigation spacecraft). In a particular case, a posteriori ephemeris of navigation spacecraft obtained by various processing centers can act as measurements (i.e. $h(R(t_{k+1})) = r(t_{k+1})$).

Suppose that

- the random vectors $w(t)$ and $\nu(t_{k+1})$ form white Gaussian noises with unknown covariance matrices of system and measurements noises

$$E[w(t)] = 0,\ E[w(t)w^T(\tau)] = Q_w(t)\delta(t-\tau);$$

$$E[\nu(t_{k+1})] = 0,\ E[\nu(t_{k+1})\nu^T(t_{i+1})] = Q_\nu(t_{k+1})\delta_{k,i};$$
$$E[\nu(t_{k+1})w^T(\tau)] = 0,\quad k,i = 0,1,\ldots,N-1,\quad \tau\in[t_0,t_N];$$

- the state vector $R(t)$ in the moment t_0 is normally distributed with the parameters

$$E[R(t_0)] = \bar{R}(t_0),\quad E[(R(t_0)-\bar{R}(t_0))(R(t_0)-\bar{R}(t_0))^T] = P(t_0)$$

and has no correlation with $w(t), \nu(t_{k+1})$ for values of k.

A mathematical description of each of the forces affecting on a satellite can be found in, for example, [23,24]. It is important to note that some of these force models include parameters, which numerical values are only partially known.

In the formation of model (1), (2) it remains problematic to take into account perturbations from solar radiation pressure on the satellite [24–29]. To compute $g_3(r(t),\dot{r}(t),\theta)$ in an inertial coordinate system, the following solar radiation prediction (SRP) model is used [17,23,29]:

$$g_3(r(t),\dot{r}(t),\theta) = \Lambda(r(t))\cdot\rho^{-2}(r(t))\cdot[i_1\cdot(\theta_1+\theta_2\cos\sigma(r(t),\dot{r}(t))+\theta_3\sin\sigma(r(t),\dot{r}(t)))+$$
$$+i_2(\theta_4+\theta_5\cos\sigma(r(t),\dot{r}(t))+\theta_6\sin\sigma(r(t),\dot{r}(t)))+$$
$$+i_3(\theta_7+\theta_8\cos\sigma(r(t),\dot{r}(t))+\theta_9\sin\sigma(r(t),\dot{r}(t)))]. \tag{3}$$

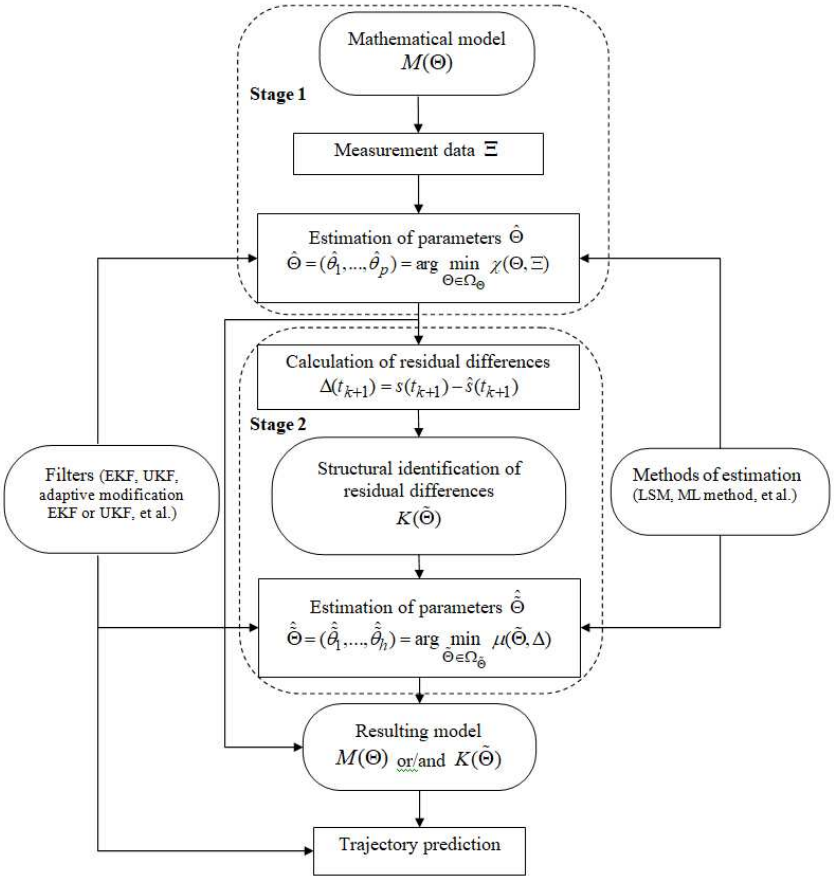
Here $\Lambda(r(t))$ is the eclipse factor, $\rho(r(t))$ is the distance between the satellite and the Sun, $\sigma(r(t),\dot{r}(t))$ is the argument of the latitude for the navigation satellite, $i_1 = \frac{r_S(t)-r(t)}{\|r_S(t)-r(t)\|}$ is ort in the direction of solar radiation, $i_2 = \frac{i_1\times r(t)}{\|i_1\times r(t)\|}$ is ort normal to the Sun-satellite-Earth, $i_3 = i_1\times i_2$ is ort that complements the system to the right triple of vectors.

2. Two-Stage Parametric Identification Procedure

With informative measurement data and a suitable model structure we can get a model with fine predictive properties and able to describe the dynamics of the process. Usually, the construction of model (1), (2) consists in finding estimates of the unknown parameters of the SRP model.

In this article we propose a two-stage procedure for parametric identification of model (1), (2) (see Figure).

At the *first stage* of the procedure, the parameters of the radiation pressure model are estimated using the ML method based on several adaptive unscented Kalman filter. The estimation of unknown parameters of the mathematical model is carried out according to measurement data Ξ and identification method. The a priori assumptions allow to use the ML method for the parameters estimation. In mild conditions, ML method estimates have such practically significant properties as asymptotic unbiasedness, consistency, asymptotic efficiency and asymptotic normality. One class of the methods to solve the problem of nonlinear models identification basically the continuous-discrete EKF applied to a linearized system. As noted earlier, the use of continuous-discrete EKF requires differentiability of nonlinear functions that belong to the right side of the equations of state, and measurement for model (1), (2) carries additional computational complexity. UKF is based on unscented transformation used for statistics computation of a random vector undergoing transformation by a nonlinear function. Unscented transformation describes statistical properties of the vector being transformed with the use of a finite set of sigma points. The set of sigma samples is chosen deterministically so that statistical moments of the first and second order of the original distribution are projected in a complete way. Next these points are transformed via nonlinear process to the new space where they describe statistics of the distribution being transformed. On the base of them output statistics are calculated. In practice, when estimating the parameters of the SRP model, it is difficult to obtain exact unknown noise statistics information which may induce large state estimation errors and filter divergence. To overcome this defect, a number of adaptive filter methods were published to weaken the impact of uncertainly noise information, and so the adaptation of noise covariance matrices becomes an important direction for developing stability and convergence filter. We propose using the multiple adaptive modifications of the UKF, which allow using a weighted average estimate of the state vector.



Scheme of the two-stage identification procedure

At the *second stage*, the parameters of the unaccounted perturbations model are estimated based on the results of measurements of residual differences. The scheme of the two-stage identification procedure is presented in Figure.

We present a two-stage algorithm for estimating the parameters of the SRP model and predicting the satellite orbital motion.

Stage 1. Construct the matching model (one-stage parametric identification)

1. Solve the problem of parametric identification based on the ML method

$$\hat{\theta} = \underset{\theta \in \Omega_\theta}{\operatorname{argmin}} \quad \frac{1}{2} \sum_{k=0}^{N-1} \ln \det P_Y(t_{k+1}) + \frac{1}{2} \sum_{k=0}^{N-1} \varepsilon(t_{k+1})^T P_Y^{-1}(t_{k+1}) \varepsilon(t_{k+1}), \tag{4}$$

where $\varepsilon(t_{k+1})$ and $P_Y(t_{k+1})$ are defined based on the corresponding equations of the following adaptive UKF [16,17].

Initialization is implemented as follows.

• Set the values

$$\xi = 0,001,\ \eta = 2,\ \phi = k = 0,\ b = 0,998.$$

• Define the initial values

$$\hat{R}^1(t_0|t_0) = \hat{R}^2(t_0|t_0) = \bar{R}(t_0),\ P(t_0|t_0) = P(t_0),\ Q_w(t_0),\ Q_\nu(t_1),$$

$$N_R, N_Q \text{ are positive numbers.}$$

• Calculate

$$l = \xi^2(n+\phi) - n\ (n = 6 \text{ is the size of } R(t)),\ \alpha_0 = \frac{l}{n+l},$$

$$\beta_0 = \frac{l}{(n+l) + (1-\xi^2+\eta)},\ \alpha_i = \frac{1}{2(n+l)} = \beta_i,\ i = 1, \ldots, 2n,$$

$$a = [\alpha_0, \alpha_1, \ldots, \alpha_{2n}]^T,$$

$$A = (I - \underbrace{[a|\ldots|a]}_{2n+1}) \cdot diag(\beta_0, \beta_1, \ldots, \beta_{2n}) \cdot (I - \underbrace{[a|\ldots|a]}_{2n+1})^T.$$

For $k = \overline{0, N-1}$

Prediction is implemented as follows.

• Define $\hat{R}(t_{k+1}|t_k)$ and $P(t_{k+1}|t_k)$ as the result of differential equations (5), (6) integration

$$\frac{d}{dt}\hat{R}(t|t_k) = R_f(t|t_k)a,\quad t_k \le t \le t_{k+1}, \tag{5}$$

$$\frac{d}{dt}P(t|t_k) = R^\tau(t|t_k)AR_f^T(t|t_k) + R_f(t|t_k)A(R^\tau(t|t_k))^T + Q_w(t),\quad t_k \le t \le t_{k+1}, \tag{6}$$

where the transformed set of vectors is defined as

$$R_f(t|t_k) = [f(R_0^\tau(t|t_k))|f(R_1^\tau(t|t_k)|\ldots|f(R_{2n}^\tau(t|t_k))]_{n\times(2n+1)},$$

the sigma points $R_i^\tau(t|t_k),\ i = \overline{1,n}$ are computed by the following formula

$$R_i^\tau(t|t_k) = \begin{cases} \hat{R}(t|t_k), & i = 0, \\ \hat{R}(t|t_k) + \sqrt{n+l}D_i(t|t_k), & i = \overline{1,n}, \\ \hat{R}(t|t_k) - \sqrt{n+l}D_{i-n}(t|t_k), & i = \overline{n+1, 2n}, \end{cases} \tag{7}$$

$$R^\tau(t|t_k) = [R_0^\tau(t|t_k)|R_1^\tau(t|t_k)|\ldots|R_{2n}^\tau(t|t_k)]_{n\times(2n+1)},$$

where D_i is the i-th row of the lower triangular matrix obtained by the Cholesky decomposition of $P(t|t_k)$.

Updating is implemented as follows.

• Find the set $R^\tau(t_{k+1}|t_k)$ using (7) with the substitution $t = t_{k+1}$.
• Calculate

$$S_h(t_{k+1}|t_k) = [h(R_0^\tau(t_{k+1}|t_k))|h(R_1^\tau(t_{k+1}|t_k)|\ldots|h(R_{2n}^\tau(t_{k+1}|t_k))]_{m\times(2n+1)},$$

$$\varepsilon(t_{k+1}) = s(t_{k+1}) - S_h(t_{k+1}|t_k)a.$$

Adaptation block I	Adaptation block II
Estimation of the covariance matrix of measurement noise	
$\bar{b}_k = \dfrac{1-b}{1-b^{k+1}},$ $\hat{Q}_\nu(t_{k+1}) = (1-\bar{b}_k)\hat{Q}_\nu(t_k)+$ $+\bar{b}_k[\varepsilon(t_{k+1})\varepsilon^T(t_{k+1})-$ $-\sum_{i=0}^{2n}\beta_i(h(R_i^\tau(t_{k+1}\|t_k)) - S_h(t_{k+1}\|t_k)a)\cdot$ $\cdot(h(R_i^\tau(t_{k+1}\|t_k)) - S_h(t_{k+1}\|t_k)a)^T]$	$\bar{\varepsilon}(t_{k+1}) = \dfrac{N_R-1}{N_R}\bar{\varepsilon}(t_k) + \dfrac{1}{N_R}\varepsilon(t_{k+1}),$ $\Delta\hat{Q}_\nu(t_{k+1}) = \dfrac{1}{N_R-1}(\varepsilon(t_{k+1}) - \bar{\varepsilon}(t_{k+1}))\cdot$ $\cdot(\varepsilon(t_{k+1}) - \bar{\varepsilon}(t_{k+1}))^T-$ $-\dfrac{1}{N_R}S_h(t_{k+1}\|t_k)AS_h^T(t_{k+1}\|t_k),$ $\hat{Q}_\nu(t_{k+1}) = \|\, diag\left(\dfrac{N_R-1}{N_R}\hat{Q}_\nu(t_k)\right)+$ $+\Delta\hat{Q}_\nu(t_{k+1})\|$
Calculation of the estimation of the state vector and the covariance matrix of estimation errors	
$P_S(t_{k+1}) = S_h(t_{k+1}\|t_k)AS_h^T(t_{k+1}\|t_k) + Q_\nu(t_{k+1}),$ $P_{RS}(t_{k+1}) = R^\tau(t_{k+1}\|t_k)AS_h^T(t_{k+1}\|t_k),$ $K(t_{k+1}) = P_{RS}(t_{k+1})P_S^{-1}(t_{k+1}),$ $\hat{R}(t_{k+1}\|t_{k+1}) = \hat{R}(t_{k+1}\|t_k) + K(t_{k+1})\varepsilon(t_{k+1}),$ $P(t_{k+1}\|t_{k+1}) = P(t_{k+1}\|t_k) - K(t_{k+1})P_S(t_{k+1})K^T(t_{k+1})$	
Estimation of the covariance matrix of the system noise	
$\hat{Q}_w(t_{k+1}) = (1-\bar{b}_k)\hat{Q}_w(t_k)+$ $+\{\bar{b}_k[K(t_{k+1})\varepsilon(t_{k+1})\varepsilon^T(t_{k+1})K^T(t_{k+1})+$ $P(t_{k+1}\|t_{k+1}) - \sum_{i=0}^{2n}\beta_i(f(R_i^\tau(t_{k+1}\|t_k))-$ $-\hat{R}(t_{k+1}\|t_k))(f(R_i^\tau(t_{k+1}\|t_k))-$ $-\hat{R}(t_{k+1}\|t_k))^T]\}$	$\vartheta(t_{k+1}) = \hat{R}(t_{k+1}\|t_k) - R_f(t_{k+1}\|t_k)a,$ $\bar{\vartheta}(t_{k+1}) = \dfrac{N_Q-1}{N_Q}\bar{\vartheta}(t_k) + \dfrac{1}{N_Q}\vartheta(t_{k+1}),$ $\Delta\hat{Q}_w(t_{k+1}) = \dfrac{1}{N_Q-1}(\vartheta(t_{k+1}) - \bar{\vartheta}(t_{k+1}))\cdot$ $\cdot(\vartheta(t_{k+1}) - \bar{\vartheta}(t_{k+1}))^T + \dfrac{1}{N_Q}P(t_{k+1}\|t_k)_N-$ $-\dfrac{1}{N_Q}(R^\tau(t_k\|t_k)AR_f^T(t_k\|t_k)+$ $+R_f(t_k\|t_k)A(R^\tau)^T(t_k\|t_k))_N,$ $\hat{Q}_w(t_{k+1}) = \|\, diag\left(\dfrac{N_Q-1}{N_Q}\hat{Q}_w(t_k)\right)+$ $+\Delta\hat{Q}_w(t_{k+1})\|$

Calculation of the combined estimation of the state vector and the covariance matrix of estimation errors
$\hat{R}(t_{k+1}\|t_{k+1}) = ((P^I(t_{k+1}\|t_{k+1}))^{-1} + (P^{II}(t_{k+1}\|t_{k+1}))^{-1})^{-1}((P^I(t_{k+1}\|t_{k+1}))^{-1}\hat{R}^I(t_{k+1}\|t_k) + (P^{II}(t_{k+1}\|t_{k+1}))^{-1}\hat{R}^{II}(t_{k+1}\|t_k))$, $P^I(t_{k+1}\|t_{k+1}), R^I(t_{k+1}\|t_{k+1})$ and $P^{II}(t_{k+1}\|t_{k+1}), R^{II}(t_{k+1}\|t_{k+1})$ are found based on Adaptation block I and II, respectively.

Note that the cost function in (4) is known to have many local optima. Many algorithms exists for this kind of problems, for example, Newton's method and various quasi-Newton methods, which are the local ones. When using a gradient based local optimization method, the minimum found may not be global one, unless the initial data is chosen close enough to the global minimum. In general, if the obtained parameters assessments give a bad fit, there is no way to understand if the reason is either the convergence to a local minimum or the insufficient model structure. These problems, with local minima can be solved by using global optimization methods. The global optimization approach is used in this work in order to find the optima of problem (4).

Stage 2. Specify the matching model (identification of unaccounted disturbances from measurements of residual differences)

1. Calculate the residual differences $\Delta(t_{k+1})$ based on the estimates $\hat{\theta}$ obtained at Stage 1:

$$\Delta(t_{k+1}) = s(t_{k+1}) - \hat{s}_1(t_{k+1}), \quad k = 0, 1, \ldots, N-1, \tag{8}$$

where

$$\hat{s}_1(t_{k+1}) = h(\hat{R}(t_{k+1}|t_{k+1})), \tag{9}$$

$\hat{R}(t_{k+1}|t_{k+1})$ is the estimate of the state vector obtained at Stage 1.

2. Choose the following model:

$$\Delta^i(t_{k+1}) = a_0^i + a_1^i t_{k+1} + a_2^i t_{k+1}^2 + b_1^i \cos\left(\frac{2\pi t_{k+1}}{T}\right) + b_2^i \sin\left(\frac{2\pi t_{k+1}}{T}\right) +$$
$$+ c_1^i \cos\left(\frac{4\pi t_{k+1}}{T}\right) + c_2^i \sin\left(\frac{4\pi t_{k+1}}{T}\right), \tag{10}$$

where $i = 1, \ldots, m$, m is the size of $s(t_{k+1})$ (here $m = 3$), T is the defined value, and estimate the unknown parameters $\tilde{\theta} = (a_0^i, a_1^i, a_2^i, b_1^i, b_2^i, c_1^i, c_2^i)^T$, $i = 1, 2, 3$ using the least squares method [30]:

$$\hat{\tilde{\theta}} = (B^T B)^{-1} B^T \Delta(t_{k+1}),$$

where $B = \begin{bmatrix} 1 & t_1 & t_1^2 & \cos\left(\frac{2\pi t_1}{T}\right) & \sin\left(\frac{2\pi t_1}{T}\right) & \cos\left(\frac{4\pi t_1}{T}\right) & \sin\left(\frac{2\pi t_1}{T}\right) \\ \ldots & \ldots & \ldots & \ldots & \ldots & \ldots & \ldots \\ 1 & t_N & t_N^2 & \cos\left(\frac{2\pi t_N}{T}\right) & \sin\left(\frac{2\pi t_N}{T}\right) & \cos\left(\frac{4\pi t_N}{T}\right) & \sin\left(\frac{4\pi t_N}{T}\right) \end{bmatrix}$.

3. Calculate $\hat{\Delta}(t_{k+1})$, $k = 0, \ldots, N-1$, taking into account the estimates $\hat{\tilde{\theta}}$ found by equation (8).
4. Calculate

$$\hat{s}_2(t_{k+1}) = \hat{s}_1(t_{k+1}) + \hat{\Delta}(t_{k+1}), \quad k = 0, \ldots, N-1, \tag{11}$$

using $\hat{\Delta}(t_{k+1})$ and $\hat{s}_1(t_{k+1})$ found by equation (9).

3. Modelling Results

As the measurement data, we take the rapid ephemeris of the Russian Global Navigation Satellite System (GLONASS) from 01/01/2013. In this case, it takes the satellite one revolution around the Earth to make (i.e. to pass through the different light zones). At the initial time, we compute the velocity of the satellite on the basis of rapid ephemeris using Everett interpolation. Estimation of the SRP model parameters (3) can be carried out using the two-stage parametric identification procedure according to the trajectory observations in areas of total illumination and penumbra zones.

At Stage 1, estimates of the parameters of the SRP model were found using two-stage parametric identification procedure and Adaptation block I (denoted by $\hat{\theta}^I$) and Adaptation block II (denoted by $\hat{\theta}^{II}$), as well as the multiple adaptive modifications of the UKF (denoted by $\hat{\theta}$). We calculate orbits (9) $\hat{s}_1^I(t_{k+1}),\ \hat{s}_1^{II}(t_{k+1}),\ \hat{s}_1(t_{k+1}),\ k = 0, \ldots, N-1$, using adaptive modifications of UKF for the estimates $\hat{\theta}^I, \hat{\theta}^{II}, \hat{\theta}$ found.

At Stage 2, we calculate the residual differences $\Delta^I(t_{k+1}), \Delta^{II}(t_{k+1}), \Delta(t_{k+1})$, $k = 0, \ldots, N-1$, (8) from those $\hat{s}_1^I(t_{k+1}),\ \hat{s}_1^{II}(t_{k+1}),\ \hat{s}_1(t_{k+1})$ found at Stage 1. By $\Delta^I(t_{k+1}), \Delta^{II}(t_{k+1}), \Delta(t_{k+1})$ we construct model (10). We find $\hat{s}_1^I(t_{k+1}),\ \hat{s}_1^{II}(t_{k+1}),\ \hat{s}_1(t_{k+1})$ for $k = N, \ldots, N+M$ (M depends on the prediction time: 24, 12, 6 hours).

Let us construct an orbit prediction on 02/01/2013 $\hat{s}_2^I(t_{k+1}),\ \hat{s}_2^{II}(t_{k+1}),\ \hat{s}_2(t_{k+1})$ for $k = N, \ldots, N+M$ using (11).

To assess the quality of the orbit prediction using one-stage and two-stage parametric identification procedures, we calculate the root mean square (RMS) values of the difference between the predicted and final orbits for 24 hours, 12 hours, 6 hours on 02/01/2013. The results are presented in Table 1 and Table 2.

Table 1

The RMS values of orbit differences between predicted and final orbits 24-h, 12-h, 6-h on 02/01/2013 (one-stage parametric identification procedure, unit: cm)

Prediction time	Satellite	RMS		
		Radial	Along-track	Cross-track
24-h	R01	3,14	14,01	4,12
	R02	2,96	12,41	4,61
	R03	3,89	13,67	4,72
12-h	R01	2,74	11,24	3,75
	R02	2,39	11,94	3,64
	R03	3,05	12,17	3,58
6-h	R01	2,04	9,14	2,68
	R02	1,99	9,84	3,02
	R03	1,87	9,44	2,94

Two-stage parametric identification procedure based on the multiple adaptive UKF algorithm makes it possible to construct a more accurate model of satellite motion and make a prediction of satellite motion for a given dimensional interval.

Conclusion

The proposed approach to the construction of a satellite motion model and the prediction of ephemeris is based on a complex application of the maximum likelihood method, the nonlinear filtering algorithm and the identification of a complex component

Table 2

The RMS values of orbit differences between predicted and final orbits 24-h, 12-h, 6-h on 02/01/2013 (two-stage parametric identification procedure, unit: cm)

Prediction time	Satellite	Adaptation block I			Adaptation block II			Multiple adaptive UKF		
		Position error								
		Radial	Along-track	Cross-track	Radial	Along-track	Cross-track	Radial	Along-track	Cross-track
24-h	R01	2,21	12,13	3,82	2,18	11,46	3,74	1,81	10,92	3,64
	R02	2,17	11,36	4,13	2,22	11,96	4,29	1,73	10,43	4,03
	R03	3,06	12,45	3,96	2,99	12,04	3,88	2,66	10,22	3,51
12-h	R01	1,87	10,47	3,05	1,67	9,99	3,17	1,66	9,67	2,98
	R02	2,1	11,03	3,12	2,4	10,34	2,97	1,8	9,58	2,87
	R03	1,95	10,88	2,94	2,09	10,06	3,01	1,78	10,01	2,65
6-h	R01	1,11	8,46	2,19	1,12	8,07	2,17	1,03	7,87	1,89
	R02	1,23	9,12	2,67	0,99	9,54	2,73	0,94	8,99	2,11
	R03	1,19	8,66	2,12	1,04	8,53	2,46	0,89	8,23	2,12

of the satellite motion model. The use of the two-stage parametric identification procedure that combines the estimation of the parameters of the SRP model from ephemerides with the refinement of the residual accelerations made it possible to increase the accuracy of determining the unknown parameters of the satellite motion model. It is shown that the combined use of several adaptive filtering algorithms makes it possible to increase the accuracy of satellite orbit prediction.

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ДВУХЭТАПНАЯ ПРОЦЕДУРА ПАРАМЕТРИЧЕСКОЙ ИДЕНТИФИКАЦИИ МОДЕЛИ ДВИЖЕНИЯ КОСМИЧЕСКОГО АППАРАТА НА ОСНОВЕ АДАПТИВНЫХ МОДИФИКАЦИЙ СИГМА-ТОЧЕЧНОГО ФИЛЬТРА КАЛМАНА

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В работе представлена новая двухэтапная процедура параметрической идентификации модели движения центра масс космического аппарата. На первом этапе процедуры с помощью метода максимального правдоподобия оцениваются параметры модели радиационного давления, при этом построение критерия идентификации осуществляется на основе нескольких адаптивных модификаций непрерывно-дискретного сигма-точечного фильтра Калмана. На втором этапе процедуры по результатам измерений остаточных разностей строится регрессионная модель неучтенных возмущений. Полученные численные результаты приводят к значительному улучшению точности прогнозирования траектории движения космического аппарата.

Ключевые слова: нелинейная стохастическая непрерывно-дискретная система; адаптивный сигма-точечный фильтр; параметрическая идентификация; радиационное давление; модель движения космического аппарата.

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