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EXACT SOLUTIONS OF BETA-FRACTIONAL FOKAS–LENELLS EQUATION VIA SINE-COSINE METHOD

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In nonlinear plasma physics, photonics and optics, the space-time fractional nonlinear Fokas–Lenells equation associated with beta derivative has significiant applications. This equation is used in this study to provide precise solutions using the Sine-Cosine method. Furthermore, using computer software, we plot the 2D-3D figures of the obtained solutions based on the appropriate parameters. The findings indicate that the suggested technique is simple, efficient and capable of producing complete solutions to nonlinear models due to mathematical physics.

Keywords: Sine-cosine method; exact solutions; beta derivative; Fokas–Lenells equation.

Introduction

Nonlinear partial differential equations (NLPDE) are utilized in physics to simulate a wide range of natural occurrences. It is critical to seek for accurate solutions to these equations in order to characterize nonlinear physical processes ranging from gravity to fluid dynamics. In order to better comprehend these equations, scientists devised practical approaches for finding accurate solutions. Some of the solutions are as follows: the Adomian decomposition [1], extended direct algebraic [2], (G'/G)-expansion [3], the Darboux transformation [4,5], $(m+G'/G)$-expansion [6], Improved Bernoulli Sub-Equation Function Method (IBSEFM) [7,8], the Hirota method [9], Sine-Cosine method [10,11], the extended tanh method [4,9,12], improved tanh function [13], etc.

Not only nonlinear partial differential equations but also space-time fractional nonlinear equations associated with beta derivative have important applications. This work takes into account beta-fractional nonlinear Fokas–Lenells equation [14]:

$$iD_t^{\mu}u + n_1 D_{xx}^{2\alpha}u + n_2 D_t^{\mu} D_x^{\alpha}u + |u|^2(su + irD_x^{\alpha}u) = i\beta D_x^{\alpha}u + i\gamma D_x^{\alpha}(|u|^{2n}u) + iuD_x^{\alpha}|u|^{2n}, \quad (1)$$

where $0 < \mu, \alpha \leq 1, i = \sqrt{-1}$ is the imaginary unit, $u = u(x,t), x$ is the spatial coordinate and t is the temporal variable, n_1, β, n_2, δ and γ are the coefficients that represents the spatiotemporal dispersion (STD), inter-modal dispersion (IMD), group velocity dispersion (GVD), nonlinear dispersion (ND) coefficient and self-steepening perturbation term, respectively, and $iD_t^{\mu}u$ is the linear fractional temporal evolution of the pulses in the nonlinear optics. The full nonlinearity is represented by the parameter n. If $\mu = \alpha = 1$, (1) is called the original Fokas–Lenells equation [15–17]. Before we begin the solution technique, we recall the beta derivative and then give the description of the proposed method.

The fractional form of (1) was investigated by putting into several methods, such as the fractional dual-function method [18], the extended direct algebraic method [19], the simplest Riccati equation scheme [14], the extended sinh-Gordon equation expansion scheme [20], ϕ^6-model expansion method [21], etc. Moreover, optical solutions to (1) were retrieved using IBSEFM in [22].

1. Beta Derivative

In this section, we will go over some of the fundamentals of the beta derivative which will be employed in this assignment.

Assume that $h(x)$ is a function for all positive x. Then, $\beta-$ derivative of $h(x)$ is defined as [23]:

$$T^{\beta}\left(h(x)\right)=\frac{d^{\beta}h(x)}{dx^{\beta}}=\lim_{\varepsilon\to 0}\frac{h\left(x+\varepsilon\left(x+\frac{1}{\Gamma(\beta)}\right)^{1-\beta}\right)-h(x)}{\varepsilon},$$

where $0<\beta\leq 1$. The β-derivative is a generalization of the classical derivative in fractional calculus. The derivative's distinctive qualities are presented in [23]. Suppose that $m(x)$ and $n(x)$ are $\beta-$differentiable functions for all $x>0$ and $\beta\in(0,1]$. Then

$$i)\ T^{\beta}\left(\gamma_1 m(x)+\gamma_2 n(x)\right)=\gamma_1 T^{\beta}\left(m(x)\right)+\gamma_2 T^{\beta}\left(n(x)\right),\ \forall\ \gamma_1,\gamma_2\ \epsilon\mathbb{R},$$

$$ii)\ T^{\beta}\left(m(x)n(x)\right)=n(x)T^{\beta}\left(m(x)\right)+m(x)T^{\beta}\left(n(x)\right),$$

$$iii)\ T^{\beta}\left(\frac{m(x)}{n(x)}\right)=\frac{n(x)T^{\beta}\left(m(x)\right)-m(x)T^{\beta}\left(n(x)\right)}{\left(n(x)\right)^2},$$

$$iv)\ T^{\beta}\left(m(x)\right)=\left(x+\frac{1}{\Gamma\left(\beta\right)}\right)^{1-\beta}\frac{dm(x)}{dx}.$$

We may easily transform a NLPDE with β-derivative into a nonlinear ordinary differential equation of integer order thanks to these features.

2. Description of Sine-Cosine Method

The Sine-Cosine method is described in this section [10, 24]. Let

$$\omega(x,t)=\omega(\eta) \tag{2}$$

be a wave transformation, where $\eta=x-ct$. The partial differential equation shown below

$$P(\omega,\omega_t,\omega_x,\omega_{xx},...)=0 \tag{3}$$

can be transformed into an ordinary differential equation

$$Q(\omega,-c\omega',\omega',\omega'',...)=0 \tag{4}$$

by (2).Then, as long as all terms have derivatives, equation (4) is integrated and integration constants are assumed to be zeros. The solutions to (4) can be phrased as follows:

$$\omega(x,t)=\alpha\cos^{\gamma}(\mu\eta), \tag{5}$$

or

$$\omega(x,t) = \alpha \sin^{\gamma}(\mu\eta), \tag{6}$$

in which α, μ and γ will be determined, μ and c are constants, the derivatives of (5) turns

$$\begin{aligned} (\omega^n)' &= -n\gamma\mu\alpha^n cos^{n\gamma-1}(\mu\xi)\sin(\mu\eta), & (7)\\ (\omega^n)'' &= -n^2\mu^2\gamma^2\alpha^n \cos^{n\gamma}(\mu\eta) + n\mu M^2\alpha^n\gamma(n\gamma-1)\cos^{n\gamma-2}(\mu\eta), & (8) \end{aligned}$$

the derivatives of (6) are

$$\begin{aligned} (\omega^n)' &= n\gamma\mu\alpha^n sin^{n\gamma-1}(\mu\eta)\cos(\mu\eta), & (9)\\ (\omega^n)'' &= -n^2\mu^2\gamma^2\alpha^n \sin^{n\gamma}(\mu\eta) + n\mu M^2\alpha^n\gamma(n\gamma-1)\sin^{n\gamma-2}(\mu\eta). & (10) \end{aligned}$$

By using (5) – (10) to (4), we obtain a trigonometric equation based on the terms $\cos^{\gamma}(\mu\eta)$ or $\sin^{\gamma}(\mu\eta)$. Then, we determine the parameters by first balancing exponents of each pair of cosine or sine to determine γ. Following that, we collect all coefficients of the same power in $\cos^{k}(\mu\eta)$ or $\sin^{k}(\mu\eta)$, where these coefficients where these coefficients ought to vanish. We can find the coefficients from the system of algebraic equations between unknown γ, α, and μ are given and from that we obtain the coefficients.

3. Mathematical Model

Consider the complex wave transformation

$$u(x,t) = V(\xi)e^{i\varphi}, \tag{11}$$

where

$$\xi = \frac{1}{\alpha}\left(x+\frac{1}{\Gamma(\alpha)}\right)^{\alpha} - \frac{c}{\mu}\left(t+\frac{1}{\Gamma(\mu)}\right)^{\mu}, \varphi = -\frac{k}{\alpha}\left(x+\frac{1}{\Gamma(\alpha)}\right)^{\alpha} + \frac{\omega}{\mu}\left(t+\frac{1}{\Gamma(\mu)}\right)^{\mu} + \varphi_0, \tag{12}$$

c,k,ω and φ_0 represent wave velocity, frequency, wave number and the phase parameter respectively. Wave transformation (11) remodels (1) into a nonlinear equation and equating real and imaginary parts, we obtain:

$$(n_1 - cn_2)V'' + (n_2k\omega - n_1k^2 - \omega - \beta k)V + (s+rk)V^3 - k\gamma V^{2n+1} = 0, \tag{13}$$

and

$$\left(c + 2n_1k + \beta - n_2(\omega + ck) - rV^2 + (2n\gamma + \gamma + 2n\delta)V^{2n}\right)V' = 0. \tag{14}$$

Setting $n = 1$; (1), (13), (14) become [25]:

$$iD_t^{\mu}u + n_1D_{xx}^{2\alpha}u + n_2D_t^{\mu}D_x^{\alpha}u + |u|^2(su + irD_x^{\alpha}u) = i\beta D_x^{\alpha}u + i\gamma D_x^{\alpha}(|u|^2u) + iuD_x^{\alpha}|u|^2, \tag{15}$$

$$(c + 2n_1k + \beta - n_2(\omega + ck) + (3\gamma + 2\delta - r)V^2)V' = 0, \tag{16}$$

$$(n_1 - cn_2)V'' + (n_2k\omega - n_1k^2 - \omega - \beta k)V + (s + rk - k\gamma)V^3 = 0. \tag{17}$$

From (16) we have

$$r = 3\gamma + 2\delta, \quad c = \frac{\beta + 2n_1k - \omega n_2}{n_2k - 1}, \tag{18}$$

since $V^2V' \neq 0$ and $V' \neq 0$, where $n_2k \neq 1$ and β represents a coupled constraints relation between the parameters.

3.1. Application of Sine-Cosine Method

By using the Sine-Cosine method, (17) can be resolved.

3.1.1. Sine Solution

The transformation may be used to get the Sine solution to (17)

$$V(\xi) = \lambda \sin^{\chi}(\mu\xi), \tag{19}$$

where the parameters λ, μ and χ will be determined. We use (19) and the derivatives

$$\begin{aligned} V'(\xi) &= \lambda\chi\mu \sin^{\chi-1}(\mu\xi)\cos(\mu\xi), &(20)\\ V''(\xi) &= -\mu^2\chi^2\lambda \sin^{\chi}(\mu\xi) + \mu^2\lambda\chi(\chi-1)\sin^{\chi-2}(\mu\xi). &(21)\end{aligned}$$

After substitution of (19) and (21) into (17), we get

$$\begin{gathered} -(n_1 - cn_2)\mu^2\chi^2\lambda \sin^{\chi}(\mu\xi) + (n_1 - cn_2)\mu^2\lambda\chi(\chi-1)\sin^{\chi-2}(\mu\xi) + \\ +(n_2k\omega - n_1k^2 - \omega - \beta k)\lambda \sin^{\chi}(\mu\xi) + (s + rk - k\gamma)\lambda^3 \sin^{3\chi}(\mu\xi) = 0. \end{gathered} \tag{22}$$

Using the balance method, by equating the exponents of $\sin^k$, from (22) we find χ as follows:

$$\chi - 2 = 3\chi \quad \Rightarrow \quad \chi = -1. \tag{23}$$

By replacing (23) in (22), we get

$$\begin{gathered} -(n_1 - cn_2)\mu^2\lambda \sin^{-1}(\mu\xi) + 2(n_1 - cn_2)\mu^2\lambda \sin^{-3}(\mu\xi) + \\ +(n_2k\omega - n_1k^2 - \omega - \beta k)\lambda \sin^{-1}(\mu\xi) + (s + rk - k\gamma)\lambda^3 \sin^{-3}(\mu\xi) = 0. \end{gathered} \tag{24}$$

From (24) we have the system

$$\begin{aligned} \sin^{-1}(\mu\xi) &: -(n_1 - cn_2)\mu^2\lambda + (n_2k\omega - n_1k^2 - \omega - \beta k)\lambda = 0, &(25)\\ \sin^{-3}(\mu\xi) &: 2(n_1 - cn_2)\mu^2\lambda + (s + rk - k\gamma)\lambda^3 = 0. &(26)\end{aligned}$$

Solving system (26) yields

$$\lambda = \sqrt{\frac{-2(n_2k\omega - n_1k^2 - \omega - \beta k)}{s + rk - k\gamma}}, \quad \mu = \sqrt{\frac{n_2k\omega - n_1k^2 - \omega - \beta k}{n_1 - cn_2}}. \tag{27}$$

Plugging (27) into (19) and (11), we get

$$u_1(x,t) = e^{i\varphi}\sqrt{\frac{-2\left(n_2k\omega - n_1k^2 - \omega - \beta k\right)}{s + rk - k\gamma}} \sin^{-1}\left(\sqrt{\frac{n_2k\omega - n_1k^2 - \omega - \beta k}{n_1 - cn_2}}\xi\right), \quad 0 < \mu, \alpha \leq 1,$$

where $\xi = \frac{1}{\alpha}\left(x + \frac{1}{\Gamma(\alpha)}\right)^{\alpha} - \frac{c}{\mu}\left(t + \frac{1}{\Gamma(\mu)}\right)^{\mu}, \varphi = -\frac{k}{\alpha}\left(x + \frac{1}{\Gamma(\alpha)}\right)^{\alpha} + \frac{\omega}{\mu}\left(t + \frac{1}{\Gamma(\mu)}\right)^{\mu} + \varphi_0,$
$r = 3\gamma + 2\delta$ and $c = \frac{\beta + 2n_1k - \omega n_2}{n_2k - 1}$.

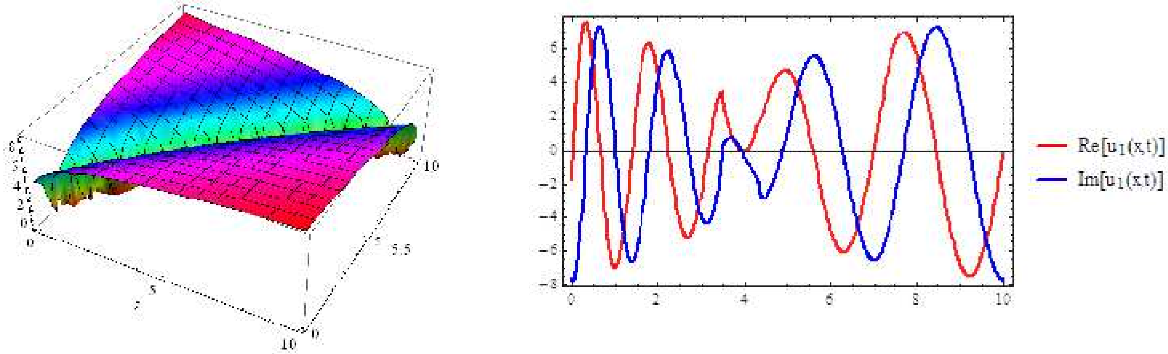

Fig. 1. 3D and 2D figures of $u_1(x,t)$ for $k = -5,1$; $\delta = -6,05$; $\alpha = 0,6$; $\eta_1 = -4,55$; $\eta_2 = -1,2$; $\beta = 1$; $\mu = 0,9$; $\omega = -1,7$; $\gamma = -1,5$; $\phi_0 = -2$; $s = -1,4$; $c = 0,7$; $t = 5,5$, $-10 < x < 10$, $-10 < t < 10$

3.1.2. Cosine Solution

The Cosine solution to (17) can be found by the transformation

$$V(\xi) = \lambda \cos^{\chi}(\mu\xi), \tag{28}$$

where the parameters λ, μ and χ will be determined. We use (28) and its derivatives

$$V'(\xi) = -\lambda\chi\mu \cos^{\chi-1}(\mu\xi)\sin(\mu\xi), \tag{29}$$

$$V''(\xi) = -\mu^2\chi^2\lambda\cos^{\chi}(\mu\xi) + \mu^2\lambda\chi(\chi-1)\cos^{\chi-2}(\mu\xi). \tag{30}$$

After substitution of (28) and (30) into (17), we obtain

$$-(n_1 - cn_2)\mu^2\chi^2\lambda\cos^{\chi}(\mu\xi) + (n_1 - cn_2)\mu^2\lambda\chi(\chi-1)\cos^{\chi-2}(\mu\xi) + \\ +(n_2k\omega - n_1k^2 - \omega - \beta k)\lambda\cos^{\chi}(\mu\xi) + (s + rk - k\gamma)\lambda^3\cos^{3\chi}(\mu\xi) = 0. \tag{31}$$

Using the balance method, by equating the exponents of $\cos^k$, from (31) we get

$$\chi - 2 = 3\chi \quad \Rightarrow \quad \chi = -1. \tag{32}$$

Substituting (32) in (31), we obtain

$$-(n_1 - cn_2)\mu^2\lambda\cos^{-1}(\mu\xi) + 2(n_1 - cn_2)\mu^2\lambda\cos^{-3}(\mu\xi) + \\ +(n_2k\omega - n_1k^2 - \omega - \beta k)\lambda\cos^{-1}(\mu\xi) + (s + rk - k\gamma)\lambda^3\cos^{-3}(\mu\xi) = 0. \tag{33}$$

From (33) we have the system

$$\cos^{-1}(\mu\xi) \ : \ -(n_1 - cn_2)\mu^2\lambda + (n_2k\omega - n_1k^2 - \omega - \beta k)\lambda = 0, \tag{34}$$

$$\cos^{-3}(\mu\xi) \ : \ 2(n_1 - cn_2)\mu^2\lambda + (s + rk - k\gamma)\lambda^3 = 0. \tag{35}$$

Solving system (35) yields

$$\lambda = \sqrt{\frac{-2(n_2k\omega - n_1k^2 - \omega - \beta k)}{s + rk - k\gamma}}, \quad \mu = \sqrt{\frac{n_2k\omega - n_1k^2 - \omega - \beta k}{n_1 - cn_2}}. \tag{36}$$

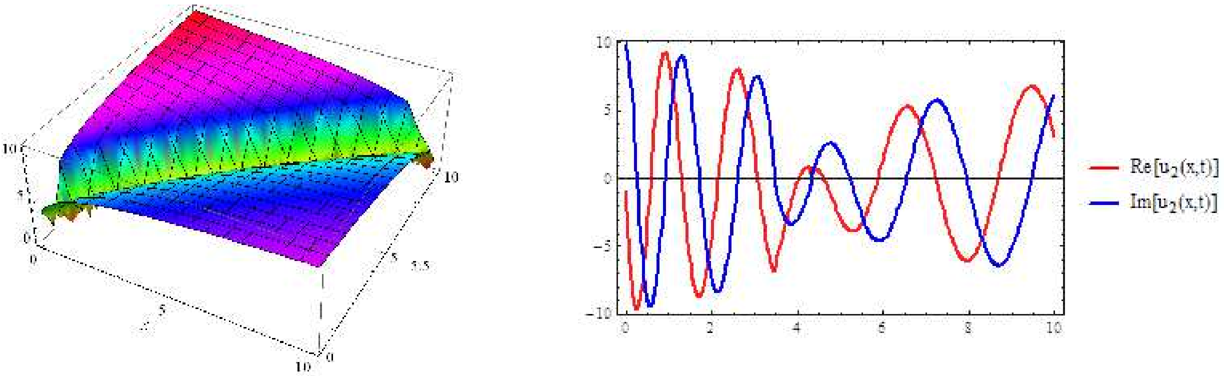

Fig. 2. 3D and 2D figures of $u_2(x,t)$ for $k = -5,1$; $\delta = -6,05$; $\alpha = 0,6$; $\eta_1 = -4,55$; $\eta_2 = -1,2$; $\beta = 1$; $\mu = 0,9$; $\omega = -1,7$; $\gamma = -1,5$; $\phi_0 = -2$; $s = -1,4$; $c = 0,7$; $t = 5,5$, $-10 < x < 10$, $-10 < t < 10$

Substituting (36) into (28) and (11) we get:

$$u_2(x,t) = e^{i\varphi}\sqrt{\frac{-2(n_2k\omega - n_1k^2 - \omega - \beta k)}{s + rk - k\gamma}}\cos^{-1}\left(\sqrt{\frac{n_2k\omega - n_1k^2 - \omega - \beta k}{n_1 - cn_2}}\xi\right), \quad 0 < \mu, \alpha \leq 1,$$

where $\xi = \frac{1}{\alpha}\left(x+\frac{1}{\Gamma(\alpha)}\right)^{\alpha} - \frac{c}{\mu}\left(t+\frac{1}{\Gamma(\mu)}\right)^{\mu}$, $\varphi = -\frac{k}{\alpha}\left(x+\frac{1}{\Gamma(\alpha)}\right)^{\alpha} + \frac{\omega}{\mu}\left(t+\frac{1}{\Gamma(\mu)}\right)^{\mu} + \varphi_0$, $r = 3\gamma + 2\delta$, $c = \dfrac{\beta + 2n_1k - \omega n_2}{n_2k - 1}$.

Conclusion

The Sine-Cosine method is employed in this article to solve a nonlinear time fractional Fokas–Lenells problem involving the beta derivative. The researched equation is turned into a nonlinear ordinary differential equation that may be solved using the proposed approach with wave transformation. Exact solutions are generated via this strategy.

The existence of the obtained solutions is verified and constraint conditions are utilized. The physical interpretation of the solutions is comparable to the form solutions in two and three dimensions. It is also evident that the more stages built, the better the approximations obtained. The findings demonstrate how easy it is to use, efficient and effective. In mathematical physics, this method can be applicated to solve a variety of nonlinear fractional partial differential equations involving in beta-derivative.

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ТОЧНЫЕ РЕШЕНИЯ БЕТА-ДРОБНОГО УРАВНЕНИЯ ФОКАСА – ЛЕНЕЛЛСА С ПОМОЩЬЮ МЕТОДА СИНУС-КОСИНУС

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В нелинейной физике плазмы, фотонике и оптике пространственно-временное дробно-нелинейное уравнение Фокаса – Ленеллса, связанное с бета-производной, имеет важные приложения. В данной работе мы рассматриваем это уравнение для построения его точных решений методом синус-косинус. Кроме того, мы строим 2D-3D фигуры полученных решений в соответствии с подходящими параметрами с помощью компьютерного программного обеспечения. Из результатов следует, что предложенный метод прост, эффективен и способен генерировать исчерпывающие решения нелинейных моделей, возникающих в математической физике.

Ключевые слова: уравнение Фокаса – Ленеллса; метод синус-косинус; бета-производная; точные решения.

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