MSC 65K10 **DOI: 10.14529/mmp240301**

GENERALIZATION OF VAPOR BUBBLE SIZE DURING UNSTEADY BOILING WITH THE USE OF TWO STAGE OPTIMIZATION METHOD

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This study aims to apply a novel technique devised by the authors to process the results of thermal physics experiments. The paper describes a two-stage technique for identifying coefficients of mathematical models from observed experimental data. The technique is based on the maximum likelihood method and is informed by the errors of all sensors used to obtain parameter measurements. Stage 1 of the technique minimizes the maximum relative error over all measured parameters, which allows gross measurement errors to be identified in qualitative terms and reduces the maximum relative error down to acceptable values. At Stage 2, we propose to use the method of weighted least absolute values to minimize the sum of absolute values of relative errors of all measured parameters. The technique was applied to process the results of thermal physics experiments aimed at generalizing the size of vapor bubbles of various types during unsteady heating of a vertical steel cylindrical heater surrounded by an upward flow of water subcooled to the saturation temperature. The numerical simulations reported in this study attest to the high quality of the proposed two-stage technique for identifying coefficients of mathematical models. The study also presents a comparative analysis of the results obtained by the classical least squares method and the novel two-stage technique.

Keywords: thermal physics experiments; coefficient identification; mathematical model; maximum likelihood criterion; weighted least absolute values method; least squares method.

Introduction

A key problem in thermal physics is how to arrive at the most faithful description of some investigated process by means of a mathematical model. In the case where there is uncertainty in the theoretical description of the process, such a mathematical model includes a number of coefficients that are meant to minimize discrepancies between modeled and empirical data. It should be noted that in some cases a significant number of individual experiments are run under different process conditions. In other cases, the number of experimental conditions is relatively small, but each of them come with a significant number of individual measurements. Each experiment measures a certain number of parameters such as temperature, pressure, flow rate, diameter, etc. The essence of the technique contributed by this study is that we measure not only the coefficients of the mathematical model but also the values of all parameters measured with some error. The errors of measurements of experimental parameters can be considered random variables with normally distributed errors, the variance and RMS errors of which are a function of the accuracy of the sensors involved in producing such measurements. Thus, the "true" value of the measured parameter falls within a certain range determined from the so-called three-sigma rule [1].

The most effective method for estimating all measured parameters and coefficients of a mathematical model is the maximum likelihood method [2]. It consists in finding such values of estimated parameters at which some likelihood function reaches its maximum. If measurement errors are random, independent, and continuous quantities, then the likelihood function for a single experiment is equal to the product of probability densities of relative errors of all measurements of the same experiment. In the case of a series of experiments, the likelihood function is equal to the sum of likelihood functions of individual experiments in the series or the average likelihood function per experiment. It is important to note that the probability density function of the normal error distribution is ill-suited to serve as an objective function for optimization problems. It consists of concave and convex parts, which leads to multiple extrema, and, even when the number of parameters to be optimized is small, this hinders the efficient optimum search. Therefore, the extreme value of another function whose extremum is close enough to the extremum of the likelihood function is what one usually looks for instead of the maximum of the likelihood function. As a rule, the sum of squares of relative errors of measurements in a series of experiments (least squares method) serves as such a function. When processing the results of thermal physics experiments, the number of parameters measured in a single experiment is small (not more than ten), but the number of individual experiments is several hundred, which leads to a significant overall number of measured parameters, exceeding a thousand. Under such conditions, the function of the sum-of-squares of relative errors, given the equality constraints as defined by the mathematical model of the process, gets a pronounced ravine-shaped profile. Therefore, this study uses the sum of absolute values of relative errors. Additional variables and inequalities [3] are introduced to yield a smooth objective function.

The physical system studied in this paper is surface boiling in heated metal channels and the measured parameter is the size of vapor bubbles. The experiments were carried out using the resources of the High-Temperature Circuit Multi-Access Research Center. The parameter is required for building predictive models of heat transfer. A widely used technique for modeling heat transfer in nucleate boiling is the heat flux partitioning [4–6]. The key bubble boiling parameter included in all components of the heat flux is the maximum bubble diameter. In the case of subcooled flow boiling, the bubble size is determined by the energy balance [7,8]. Previous published research reported a wide variety of constitutive relations for the maximum bubble diameter, relating it to the superheated layer thickness [6], heater surface superheat, and subcooling of the liquid [7,9,10]. However, the large discrepancies between the values obtained from these formulas indicate that its scope of application is extremely limited. In particular, unsteady heat release with a rapid increase in surface temperature makes all the expressions obtained for steady-state conditions yield the values of diameters that are many times higher than true values [11]. Therefore, techniques for building a mathematical model of unsteady boiling need a better consideration of the peculiarities of the extraction of empirical data obtained during the physics experiment [12,13].

1. Technique for Identifying Coefficients of the Mathematical Model of Thermal Physics Experiments

The present study employs a two-stage identification technique reported elsewhere with the contribution by the authors [3]. At Stage 1, an optimization problem is formulated and solved. The problem is to minimize the maximum relative error between the modeled and measured values over all measured parameters in all experiments. All equality constraints (equations of the mathematical model) and inequality constraints (constraints related to the RMS error of measurements) must be strictly satisfied. This stage is preliminary and is intended to identify "bad" measurements (if any) and determine the maximum relative error over all measured parameters. Relative error is equal to the absolute value of the absolute difference between observed and modeled values of the measured parameters divided by the RMS error of measurement. The mathematical statement of the optimization problem solved at this stage of the technique is as follows.

$$\min \overline{x} \tag{1}$$

subject to the following conditions:

$$x_{i1}^{r\,out} = h_1\left(x_{i1}^{r\,in}, ..., x_{iN_z^{in}}^{r\,in},\ C_1, ..., C_{N_C}\right);$$

$$..;$$

$$x_{iN_z^{out}}^{r\,out} = h_{N_z^{out}}\left(x_{i1}^{r\,in}, ..., x_{iN_z^{in}}^{r\,in},\ C_1, ..., C_{N_C}\right);$$

$$\overline{x} - \frac{x_{ij}^{r\,in} - x_{ij}^{z\,in}}{\sigma_j^{in}} \geq 0;$$

$$\overline{x} + \frac{x_{ij}^{r\,in} - x_{ij}^{z\,in}}{\sigma_j^{in}} \geq 0;$$

$$j = 1, ..., N_z^{in}$$

$$\overline{x} - \frac{x_{ik}^{r\,out} - x_{ik}^{z\,out}}{\sigma_k^{out}} \geq 0;$$

$$\overline{x} + \frac{x_{ik}^{r\,out} - x_{ik}^{z\,out}}{\sigma_k^{out}} \geq 0;$$

$$k = 1, ..., N_z^{out};$$

$$\overline{x} \geq 0;$$

$$i = 1, ..., N_e,$$

where $\overline{x}$ is an auxiliary parameter that corresponds to the maximum relative error of the measured parameters at the optimal point; $x_{ij}^{r\,in}$ is estimated value of the j-th measured input parameter of the i-th experiment; N_z^{in} is number of measured input parameters in a single experiment; $x_{ik}^{r\,out}$ is estimated value of the k-th measured output parameter of the i-th the experiment; N_z^{out} is the number of measured output parameters in a single experiment; $C_1, ..., C_{N_C}$ are coefficients of the mathematical model that are subject to optimization; N_C is the number of coefficients subject to optimization; $h_1, ..., h_{N_z^{out}}$ are expressions forming the mathematical model of the process aimed at determining the measured input parameters; $x_{ij}^{z\,in}$ is observed value of the jth measured input parameter of the ith experiment; σ_j^{in} is RMS error of the j-th measured input parameter; $x_{ik}^{z\,out}$ is observed value of the k-th measured input parameter of the i-th experiment; σ_k^{out} is RMS error of the kth measured output parameter; N_e is the number of individual experiments.

Parameters that are subject to optimization as part of this problem are auxiliary parameter $\overline{x}$; estimated measured input parameters $x_{ij}^{r\,in}$; coefficients of the mathematical model $C_1, ..., C_{N_C}$. Thus, the total number of parameters subject to optimization at Stage 1 of the technique is $N_1^{opt} = N_z^{in} \cdot N_e + N_C + 1$. Our analysis of the problem shows that the total number of inequality constraints is $N_1^{ogr\ n} = (N_z^{in} + N_z^{out}) \cdot 2 + 1$. The number of equality constraints is in turn equal to $N_1^{ogr\ r} = N_z^{out} \cdot N_e$.

Next, at Stage 2, an optimization problem is formulated and solved, which consists in minimizing the sum of absolute values of relative errors of all measured parameters in all experiments. As is known, the absolute value of some variable is a non-smooth function whose derivative has a discontinuity at zero value of the argument. In this connection, we formulate a mathematical programming problem with a smooth objective function and equality and inequality constraints so that the minimum of the sum of absolute values of relative errors of measurements is reached at its solution point. The mathematical statement of the optimization problem solved at Stage 2 of the technique can be presented as follows.

$$\min \sum_{i=1}^{N_e} \left(\sum_{j=1}^{N_z^{in}} \overline{x_{ij}^{in}} + \sum_{k=1}^{N_z^{out}} \overline{x_{ik}^{out}} \right) \tag{2}$$

subject to the following conditions:

$$x_{i1}^{r\ out} = h_1 \left(x_{i1}^{r\ in}, ..., x_{iN_z^{in}}^{r\ in},\ C_1, ..., C_{N_C} \right);$$

$$..;$$

$$x_{iN_z^{out}}^{r\ out} = h_{N_z^{out}} \left(x_{i1}^{r\ in}, ..., x_{iN_z^{in}}^{r\ in},\ C_1, ..., C_{N_C} \right);$$

$$\overline{x_{ij}^{in}} - \frac{x_{ij}^{r\ in} - x_{ij}^{z\ in}}{\sigma_j^{in}} \geq 0;$$

$$\overline{x_{ij}^{in}} + \frac{x_{ij}^{r\ in} - x_{ij}^{z\ in}}{\sigma_j^{in}} \geq 0;$$

$$0 \leq \overline{x_{ij}^{in}} \leq x^{max};$$

$$j = 1, ..., N_z^{in}$$

$$\overline{x_{ik}^{out}} - \frac{x_{ik}^{r\ out} - x_{ik}^{z\ out}}{\sigma_k^{out}} \geq 0;$$

$$\overline{x_{ik}^{out}} + \frac{x_{ik}^{r\ out} - x_{ik}^{z\ out}}{\sigma_k^{out}} \geq 0;$$

$$0 \leq \overline{x_{ik}^{out}} \leq x^{max};$$

$$k = 1, ..., N_z^{out};$$

$$i = 1, ..., N_e,$$

where the notations are the same as in the optimization problem (1); x_{ij}^{in} is an auxiliary parameter that corresponds to j-th measured input parameter of the i-th experiment; x_{ik}^{out} is estimated value of the k-th measured output parameter of the i-th the experiment; x^{max} is maximum permissible error of measured parameters, determined at the first stage of the methodology.

Parameters that are subject to optimization as part of this problem are auxiliary parameters $x_{ij}^{in}, x_{ik}^{out}$; estimated measured input parameters $x_{ij}^{r\,in}$; coefficients of the mathematical model $C_1, ..., C_{N_C}$. Thus, the total number of parameters subject to optimization at Stage 2 of the technique is: $N_2^{opt} = (N_z^{in} + N_z^{out}) \cdot N_e + N_z^{in} \cdot N_e + N_C$. Our analysis of the problem shows that the total number of inequality constraints is: $N_2^{ogr\,n} = (N_z^{in} + N_z^{out}) \cdot 6$. The number of equality constraints is in turn equal to: $N_2^{ogr\,r} = N_z^{out} \cdot N_e$.

It is important to note that the value of the maximum relative error of measurements $\overline{x}$ obtained at Stage 1 is rounded up and fixed, and then serves as an upper limit x^{max} for all relative errors at Stage 2. Thus, the estimated values of all quantities measured with error are determined at the optimal point of the solution to this problem, as well as the required coefficients of the mathematical model of the thermal physics experiment.

The likelihood function (sum of probability densities), as well as the functions of the sum of absolute values and sum of squares of relative errors of a series of experiments are determined, respectively, from the following expressions.

$$F^{pr} = \sum_{i=1}^{N_e} \left[\prod_{j=1}^{N_z^{in}} f\left(\frac{\left| x_{ij}^{r\,in} - x_{ij}^{z\,in} \right|}{\sigma_j^{in}} \right) \cdot \prod_{k=1}^{N_z^{out}} f\left(\frac{\left| x_{ik}^{r\,out} - x_{ik}^{z\,out} \right|}{\sigma_k^{out}} \right) \right]; \tag{3}$$

$$f(x) = \frac{1}{\sqrt{2\pi}} e^{\frac{-x^2}{2}};$$

$$F^{abs} = \sum_{i=1}^{N_e} \left[\sum_{j=1}^{N_z^{in}} \left(\frac{\left| x_{ij}^{r\,in} - x_{ij}^{z\,in} \right|}{\sigma_j^{in}} \right) + \sum_{k=1}^{N_z^{out}} \left(\frac{\left| x_{ik}^{r\,out} - x_{ik}^{z\,out} \right|}{\sigma_k^{out}} \right) \right]; \tag{4}$$

$$F^{sqr} = \sum_{i=1}^{N_e} \left[\sum_{j=1}^{N_z^{in}} \left(\frac{\left(x_{ij}^{r\,in} - x_{ij}^{z\,in} \right)^2}{\left(\sigma_j^{in} \right)^2} \right) + \sum_{k=1}^{N_z^{out}} \left(\frac{\left(x_{ik}^{r\,out} - x_{ik}^{z\,out} \right)^2}{\left(\sigma_k^{out} \right)^2} \right) \right]. \tag{5}$$

The above functions can be used as quality criteria for comparative analysis of different versions of mathematical models of the same thermal physics experiment. On the other hand, these functions allow us to evaluate how well different computational techniques perform when applied to the same sample of experimental data using the same mathematical models of the experiment. For example, we can perform a numerical comparison between the results obtained by the classical least squares method and the results obtained by the novel two-stage technique detailed in this paper.

2. Description of the Experimental Setup

Experimental data of unsteady nucleate boiling were obtained in a vertical channel with a vertical hollow cylindrical heater made of Type 321 stainless steel (length is 120 mm, diameter is 12 mm, thickness is 1 mm). The total heat release per unit area of the heater reached 20 MW/m^2, which is much larger than the heat flux picked up by the supercooled flow, which can be 2 MW/m^2 [6]. For this reason, we observed a rapid (up to 3000 K/s) increase in surface temperature, and evaporation occurred in a highly unsteady temperature field. This study investigates boiling at high levels of water subcooling. Depending on the level of subcooling, which was 23 K, 83 K, and 103 K, the nucleate boiling phase began at 35 ms, 30 ms, and 13 ms from the onset of heating, respectively, and ended with a transition to boiling accompanied by massive bubble fusion and the formation of large vapor structures. The heat release duration was 150–180 ms and the initial pressure in the test area was 0,29 MPa. The initial flow velocity was 0,5 m/s. To ensure degassing, water was boiled for several hours while maintaining circulation in the experimental setup. The gases produced by boiling were removed through a degassing tank that was connected to the ambient air. The degassing tank was then shut off by closing the switch valves. The boiling video was captured at up to 180 kHz and a spatial resolution of 5,5 μm per pixel at a frame size of 256×256 pixels. The errors in the measurements of bubble diameter, temperature, pressure, and bulk velocity were ± 5 μm, $\pm 0,5K$, ± 3 kPa, and $\pm 0,01$ m/s, respectively.

3. Maximum Sizes of Vapor Bubbles at Unsteady Boiling

In what follows we apply the above approach to process the results of a thermal physics experiment aimed at generalizing the size of vapor bubbles of various types during unsteady heating of a vertical steel cylindrical heater surrounded by an upward flow of water subcooled to the saturation temperature. Experimental data were obtained by taking a video capture of the experimental area at a high frame rate and then processing the video frames. We processed about 2,800 bubbles at three subcooling values ΔT_{sub} (23,1; 83; 103 K). The observed bubbles were assigned to one of three classes: single bubbles (*single*, s), clustered bubbles (*cluster*, c), and pulsating bubbles (*pulsating*, p). Moreover, all bubbles were also averaged regardless of their type, thus forming the fourth class of bubbles averaged over all bubble types (*all*, a). Each bubble is characterized by a maximum diameter D_m, lifetime t_b, and bubble type.

As evidenced by the results of experiments reported in [6, 11], bubbles of a wide variety of sizes, differing by a factor of about 10, exist on the heater surface throughout the nucleate boiling phase simultaneously, while the change in the average size over the entire nucleate boiling phase does not exceed 400%. Therefore, when building a model of nucleate boiling, the input data are not the diameters of individual bubbles, but the values averaged over a time interval with a weighting coefficient, which allows to obtain the correct value of the averaged heat fluxes in the future. Existing results on estimating the relative contribution of different components of the heat flux [6] showed that as the heater temperature increases, the heat flux contribution of the initial vapor evaporation in the bubble volume increases as well. The magnitude of this heat flux component is proportional to the diameter of the bubble raised to the power of three. Therefore, the model identification is based on D_{vol} as defined by the following relation.

$$D_{vol} = \frac{\sum_{i=1}^{N}(D_m^4 \cdot t_b)}{\sum_{i=1}^{N}(D_m^3 \cdot t_b)}, \tag{6}$$

where D_m is the maximum size of the observed bubble, N is the number of observed bubbles within the current time range, t_b is a weighting factor capturing the lifetime of the bubble within the current time range ($0 \leqslant t_b \leqslant 1$).

Averaging bubble diameters as per the formula reported in (6) was performed for time intervals equal to 0,3 ms with a step of 0,055 ms for bubbles of each type. To this end, we took into account the weighting coefficient of the bubble t_b, characterizing its lifetime within the considered time range. That is, if a bubble existed only half the time range under consideration, its weighting coefficient was 0,5.

According to several studies [7, 9–11], the maximum bubble size depends on heater wall temperature and subcooling of the liquid. The saturation temperature T_s is usually chosen as the reference point for both subcooling of the liquid and heater wall superheat. Subcooling of the liquid is defined as $\Delta T_{sub} = (T_s - T_0)$, where T_0 is the water temperature at the channel inlet. The heater wall superheat is most often defined as $(T_w - T_s)$ [7, 9–11]. However, as was shown in [6], the choice of T_{ONB} as the temperature of the first bubble appearance allows for consistency of the description of the results for very different superheat values. Depending on the subcooling in this series of experiments, the shift of these reference points, $(T_{ONB} - T_s)$, varies from 15 to 35 K, so the ranges $(T_w - T_s)$ do not overlap. Thus, the wall superheat is defined as $(T_w - T_{ONB})$, which is 0 for the first bubble. Since there is no single model relating subcooling to wall superheat to maximum bubble diameter, we chose a formula that is linear with respect to both parameters. Thus, the mathematical model of the weighted average diameter for different types of bubbles has the following form.

$$D_{vol,s} = [(A_s \cdot \Delta T_{sub} + B_s) \cdot (T_w - T_{ONB}) + C_s \cdot \Delta T_{sub} + D_s)] \cdot 10^{-5}; \qquad (7)$$

$$D_{vol,c} = [(A_c \cdot \Delta T_{sub} + B_c) \cdot (T_w - T_{ONB}) + C_c \cdot \Delta T_{sub} + D_c)] \cdot 10^{-5}; \qquad (8)$$

$$D_{vol,p} = [(A_p \cdot \Delta T_{sub} + B_p) \cdot (T_w - T_{ONB}) + C_p \cdot \Delta T_{sub} + D_p)] \cdot 10^{-5}; \qquad (9)$$

$$D_{vol,a} = [(A_a \cdot \Delta T_{sub} + B_a) \cdot (T_w - T_{ONB}) + C_a \cdot \Delta T_{sub} + D_a)] \cdot 10^{-5}, \qquad (10)$$

where A, B, C, D are the estimated (adjustable) coefficients of the mathematical model of a thermophysical process; the subscript s refers to single bubbles (*single*), the subscript c refers to clustered bubbles (*cluster*), the subscript p refers to pulsating bubbles (*pulsating*), and the subscript a refers to bubbles averaged over all bubble types (*all*). Here D_{vol} is measured in millimeters.

4. Optimization Results

The two-stage technique for processing experimental data of thermal physics experiments implies estimation of all parameters measured with an error falling within the specified accuracy range of measuring instruments. Thus, the RMS errors were as follows: for the value of subcooling ΔT_{sub} – 0,1 K, for the value of the independent variable $(T_w - T_{ONB})$ – 0,5 K, for the weighted average bubble diameter D_{vol} – 5 μm. It should be noted that we assumed that the independent variable $(T_w - T_{ONB})$ changed withing the range of 3 to 14 K to adjust the coefficients of the mathematical model and to estimate the values of the measured parameters. This was done to exclude the starting area with abnormally high values of bubble diameters.

A total of 502 experiments were processed; they were organized into three scenarios (experimental conditions) based on different subcooling values: 164 experiments at ΔT_{sub} = 103 K; 117 experiments at ΔT_{sub} = 83 K; 221 experiments at ΔT_{sub} = 23,1 K. It is worth noting that no pulsating bubbles were observed under experimental conditions with ΔT_{sub} = 83 K. Each experiment contained 2 estimated input parameters and 1 estimated output parameter.

The optimization problem formulated for Stage 1 of solving the problem included 502 equality constraints organized into three scenarios based on different subcooling values ΔT_{sub} (23,1, 83, 103 K). Under each of these scenarios we observed that the change in the weighted average bubble diameter D_{vol} depended on the change in the independent variable ($T_w - T_{ONB}$) across different bubble types. Furthermore, the optimization problem contained two inequality constraints for each measured parameter. Thus, the number of individual estimated measured parameters was $N_{par} = 502 \cdot 3 = 1,506$ (of which 1,004 were input parameters for the mathematical model and 502 were output parameters).

The dimensionality of the optimization problem at Stage 1 of the technique at the end amounted to 1,021 parameters subject to optimization (including 1,004 input parameters ΔT_{sub} and ($T_w - T_{ONB}$), 16 adjustable coefficients, 4 for each of the bubble types, and 1 additional parameter characterizing the maximum relative error of the measured parameters $\overline{x}$); 502 equality constraints (one for each of experiments), 3,012 inequality constraints (2 inequality constraints for each of 1,506 estimated measured parameters). At the optimal point of the solution at Stage 1, we obtained the value of the maximum relative error, which was 6,53 sigma. The sum of the probability densities for all experiments (likelihood function) was 6,4, or an average of 0,0128 per experiment. The sum of the relative error absolute values was 2,086 or an average of 1,4 per measurement. The sum of squares of the relative errors was 6,781 and the RMS error was 2,12 per measurement.

At Stage 2 of solving the problem, we arrived at the value of the minimum sum of absolute values of relative errors of all measured parameters, which amounted to 1,449, or an average of 0,96 per measurement (the improvement with respect to this criterion was 30% compared to the value obtained after Stage 1 of the technique). This was done within the limits of the maximum relative error x^{max} obtained after Stage 1, which was fixed at 6,60. At the same time, the sum of probability densities for all experiments (the likelihood function) increased significantly and amounted to 9,9, or an average of 0,0198 per experiment (the improvement with respect to this criterion amounted to 36% compared to the value obtained after Stage 1 of the technique). The sum of squares of relative errors decreased by 3% compared with Stage 1 of the technique and amounted to 6,382 and the RMS error was 2,06 per measurement.

The dimensionality of the optimization problem at Stage 2 was as follows: 2,526 parameters subject to optimization (of which 1,004 were input parameters ΔT_{sub} and ($T_w - T_{ONB}$), 16 adjustable coefficients, 4 for each of the bubble types, and 1,506 auxiliary parameters, one for each estimated measured parameter, the sum of which makes up the objective function of the optimization problem); 502 equality constraints (one per experiment), 6,024 inequality constraints (2 inequality constraints per each of the 1,506 estimated measured parameters and 2 inequality constraints per each auxiliary parameter). At the optimal point of the solution, we obtained the values of the estimated measured parameters and the values of the adjusted coefficients of the mathematical model.

Table 1 shows the coefficients obtained for different types of bubbles by the classical least squares method and the proposed two-stage technique. Figs. 1–3 show the observed values of weighted average bubble diameters, as well as linear relationships plotted with the obtained coefficients for bubbles of different types under three scenarios of subcooling of the liquid. Single bubbles are shown in red, clusters are shown in yellow, pulsating bubbles are in green, and blue stands for the bubbles averaged over all types (*all*). The linear relationship plotted with the dotted line uses the coefficients obtained by the least squares method, the dashed line indicates the linear relationship that uses the coefficients obtained after Stage 1 of the technique, and the solid line plots the linear relationship based on the coefficients obtained after Stage 2 of the technique.

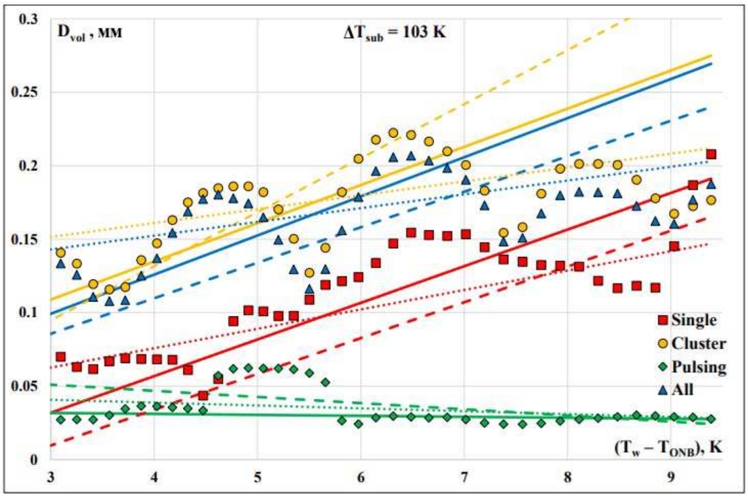

Fig. 1. Initial measurements and obtained linear relationships at $\Delta T_{sub} = 103\ K$

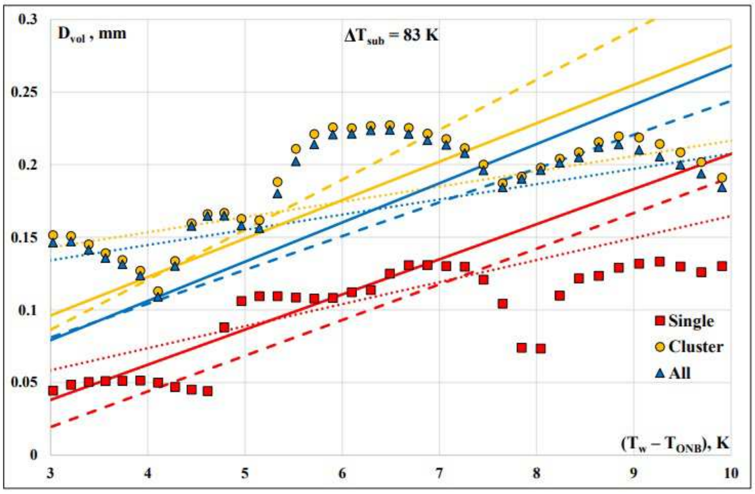

Fig. 2. Initial measurements and obtained linear relationships at $\Delta T_{sub} = 83\ K$

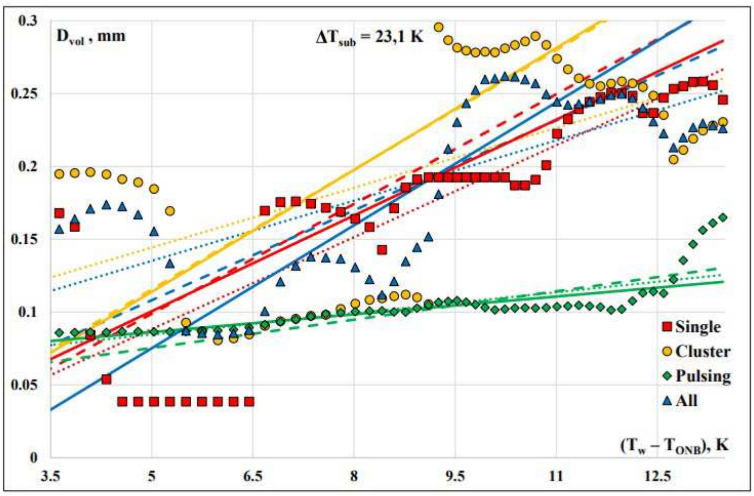

Fig. 3. Initial measurements and obtained linear relationships at $\Delta T_{sub} = 23{,}1\ K$

Table 1

Values of obtained coefficients of the mathematical model of the process

Bubble types	Single	Cluster	Pulsing	Averaged (*all*)
Coefficients obtained by the least squares method				
Coefficient A	$-9,8$	$-5,3$	$-8,5$	$-5,5$
Coefficient B	2330	1492	679	1505
Coefficient C	49,9	59,3	$-17,3$	60,9
Coefficient D	-2844	6214	6443	5218
Coefficients obtained after Stage 1 of the technique				
Coefficient A	$-0,9$	11,7	$-13,4$	4,5
Coefficient B	2534	2469	957	1955
Coefficient C	$-45,9$	7,5	25,8	10,0
Coefficient D	-1618	-2321	3697	292
Coefficients obtained after Stage 2 of the technique				
Coefficient A	3,8	$-2,4$	$-5,9$	$-1,9$
Coefficient B	2105	2845	544	2858
Coefficient C	$-42,4$	70,8	$-40,2$	106,3
Coefficient D	73	-4198	7518	-9009

Table 2

Values of quality criteria of the performed identification of coefficients

Calculation method	Least squares method	After Stage 1 of the technique	After Stage 2 of the technique
Maximum relative error, σ	23,24	6,53	6,60
Sum of absolute values of relative errors, σ	2483	2086	1449
Average relative error per measurement, σ	1,65	1,39	0,96
Sum of squares of relative errors, σ^2	22328	6781	6382
RMS error per measurement, σ	3,85	2,12	2,06
Sum of probability densities	6,9	6,4	9,9
Average probability density per experiment	0,0138	0,0128	0,0198

Table 2 sums up the values of the quality criteria for the performed identification of coefficients of the mathematical model of thermal physics experiments. The analysis of these criteria allows us to compare numerically the classical least squares method and the novel two-stage technique detailed in this paper. The least squares method has an excessively high maximum relative error value of over 23 sigma. This can be explained by the fact that this method considers all input parameters as true values of the measured parameters, and the errors of sensors measuring the corresponding parameters are ignored. As a result, the sum of relative error absolute values (4) and the sum of relative error squares (5) are larger than the value obtained after the Stage 2 of our technique by 42 % and 72 %, respectively. The sum of probability densities (3) is in turn 30 % higher (i.e., better) in the case of the technique contributed by this study. All things considered, this attests to the high quality of the proposed two-stage technique for identifying coefficients of mathematical models.

The above optimization problems for Stage 1 and Stage 2 of the technique for processing experimental data of thermal physics experiments were solved by the stepwise optimization method developed at the Melentiev Energy Systems Institute, SB RAS [14]. The method has a high solution accuracy and has been tested in identification and optimization of mathematical models of complex cogeneration plants [15, 16].

Conclusion

This paper contributed a novel two-stage technique for processing the results of thermal physics experiments. The technique aims at identifying coefficients of mathematical models of experiments from observed experimental data. The technique is based on the maximum likelihood method and is informed by the errors of all sensors used to obtain parameter measurements. Stage 1 of the technique involves minimization of the maximum relative error over all measured parameters, which allows gross measurement errors to be identified and reduces the maximum relative error down to acceptable values. At Stage 2, we rely on the method of weighted least absolute values to minimize the sum of absolute values of relative errors of all measured parameters. The technique was applied to process the results of thermal physics experiments aimed at generalizing the size of vapor bubbles of various types during unsteady heating at the surface of a steel heater in the flow of water subcooled to the saturation temperature. As is evidenced by our comparison of the quality of identification of model parameters, the technique reported in this study yields 1) lower values of criteria measuring the discrepancy from experimentally observed values and 2) higher probability density values than the least squares method used previously.

Acknowledgements. *The research was funded by the Russian Science Foundation, Grant No. 23-29-00628.* URL: https://rscf.ru/project/23-29-00628/.

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Received May 6, 2024

УДК 51.72+536.423 **DOI: 10.14529/mmp240301**

ОБОБЩЕНИЕ РАЗМЕРА ПАРОВЫХ ПУЗЫРЬКОВ ПРИ НЕСТАЦИОНАРНОМ КИПЕНИИ С ПРИМЕНЕНИЕМ ДВУХЭТАПНОЙ МЕТОДИКИ ОПТИМИЗАЦИИ

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Целью исследования является применение разработанной авторами оригинальной методики обработки результатов теплофизических экспериментов. В статье приводится описание двухэтапной методики идентификации коэффициентов математических моделей по результатам замеренных опытных данных. Методика основана на методе максимального правдоподобия и учитывает погрешности всех датчиков, используемых для получения замеряемых параметров. На первом этапе решения задачи методика предполагает минимизацию максимальной относительной погрешности среди всех замеряемых параметров, что позволяет качественно выявлять грубые погрешности измерений и снижать максимальную относительную погрешность до приемлемых значений. На втором этапе предлагается применять метод взвешенных наименьших модулей для минимизации суммы модулей относительных погрешностей всех замеряемых параметров. Данная методика была применена для обработки результатов теплофизических экспериментов, направленных на обобщение размера паровых пузырьков различных типов при нестационарном нагреве вертикально расположенного стального цилиндрического нагревателя, омываемого восходящим потоком воды, недогретой до температуры насыщения. Вычислительные эксперименты, представленные в данной работе, демонстрируют высокое качество предлагаемой двухэтапной методики идентификации коэффициентов математических моделей. В работе также представлено сравнение результатов, полученных классическим методом наименьших квадратов и предлагаемой двухэтапной методикой.

Ключевые слова: теплофизический эксперимент; идентификация коэффициентов; математическая модель; критерий максимального правдоподобия; метод взвешенных наименьших модулей; метод наименьших квадратов.

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Поступила в редакцию 6 мая 2024 г.