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ON THE METHOD OF NUMERICAL SIMULATION OF LIMIT REACHABLE SETS FOR LINEAR DISCRETE-TIME SYSTEMS WITH BOUNDED CONTROL

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The paper considers the issues of numerical modeling of the limit reachable sets for linear discrete-time systems with convex control constraints. The method based on the principle of contraction mappings has been developed. This method is designed to construct an external estimate of the limit reachable set, which is a significant problem in control theory and analysis of dynamical systems. The application of the principle of contraction mappings makes it possible to obtain an estimate with an arbitrary order of accuracy in the sense of the Hausdorff distance. Moreover, the limit point up to the closure must coincide with the limit reachable set. The value of the compression ratio depends on the choice of the norm in the vector space, which, accordingly, influences the Hausdorff distance in the compact space, as well as the operator norm of the system matrix. To demonstrate the capabilities of the proposed method, a three-dimensional system with real eigenvalues is presented as an example. Additionally, an example for constructing the limit reachable set in the damping system of a high-rise structure located in a seismic zone is provided.

Keywords: linear discrete-time system; limit reachable set; contraction mapping; convex set; polyhedron estimation.

Introduction

When solving control problems in dynamic systems, it is often necessary to take into account various limitations associated with the technical aspects of the system under study. Such limitations lead to the fact that the system can be transferred from a given initial state to a bounded set of terminal states even with an infinite time horizon. This fact makes it relevant to study not only the issues of reachability and controllability of various dynamic systems, but also the development of methods for constructing and estimating limit reachable and controllable sets for an arbitrary control system. In addition, controllable and reachable sets can be used in a number of optimal control problems to form positional control [1] for discrete-time systems.

At the moment, two main directions can be distinguished on this topic: the study of individual states for controllability [2–5] and geometric methods for constructing controllable and reachable sets [6–9]. Thus, in the study of nonlinear systems, it is possible to obtain only general properties of controllability sets [2] or their estimates [8, 9]. For the case of linear equations of dynamics in terms of state and control, it turns out to be possible to construct more constructive results for various classes of systems: periodic [10], switchable [3], with positive control [6]. The most rigorous results are formulated for the case of compact and convex constraints on control values [1, 7], which even allow the description of limit reachable and controllable sets [4, 5, 11]. In [12], for linear discrete-time systems with scalar control, on which a total 1st-order constraint is imposed, it is shown that in the case of stable systems it is possible to explicitly find the limit reachable set, which is a convex polyhedron symmetric with respect to zero. For higher-order constraints, the description of the limit reachable and 0-controllable sets is obtained by using the reference half-spaces [13].

A significant disadvantage of these methods is the inability to determine the accuracy of the estimates in advance. This article examines the development of a fundamentally new approach to numerical modeling of limit reachable sets based on the principle of contraction mappings, which was proposed in [11]. It is proved that the closure of the limit set of attainability is a fixed point of the contraction mapping given in Hausdorff space. This allows using the simple iteration method to approximate the limit reachable set with any predetermined accuracy.

1. Problem Formulation

We consider an n-dimensional линейная autonomous discrete control system $(A,\mathcal{U})$ with bounded control:

$$\begin{gathered} x(k+1) = Ax(k) + u(k), \\ x(0) = x_0,\ u(k) \in \mathcal{U},\ k \in \mathbb{N} \cup \{0\}, \end{gathered} \tag{1}$$

where $x(k), u(k) \in \mathbb{R}^n$ are vectors of state and control, respectively, $\mathcal{U} \subset \mathbb{R}^n$ is a convex compact set of admissible control values, $A \in \mathbb{R}^{n\times n}$ is the matrix of the system. We take an assumption that $0 \in \operatorname{int} \mathcal{U}$.

We denote a family of reachable sets by $\{\mathcal{Y}(N)\}_{N=0}^{\infty}$, where each $\mathcal{Y}(N)$ represents a set of those states into which, by choosing an admissible control, the (1) system can be translated from the origin in N steps:

$$\mathcal{Y}(N) = \begin{cases} \{x \in \mathbb{R}^n \colon \exists u(0), \ldots, u(N-1) \in \mathcal{U} \colon x(0) = 0, x(N) = x\}, & N \in \mathbb{N}, \\ \{0\}, & N = 0. \end{cases} \tag{2}$$

It is required to construct the limit reachable set $\mathcal{Y}_\infty$, i.e. the set of those states into which the system $(A,\mathcal{U})$ can be translated from the origin in any finite number of steps:

$$\mathcal{Y}_\infty = \{x \in \mathbb{R}^n \colon \exists N \in \mathbb{N},\ \exists u(0), \ldots, u(N-1) \in \mathcal{U} \colon x(N) = x, x(0) = 0\}.$$

Taking into account (2), the identity is also true

$$\mathcal{Y}_\infty = \bigcup_{N=0}^{\infty} \mathcal{Y}(N). \tag{3}$$

2. External Estimation of the Limit Reachable Set Based on the Principle of Contraction Mappings

We formulate the basic property of the limit reachable sets of the 1 system in the form of the following theorem:

Theorem 1. *For each n-dimensional system $(A,\mathcal{U})$ of the form (1) it is true that $\mathcal{Y}_\infty$ is a convex set.*

Proof. Let x^1, x^2 be $\in \mathcal{Y}_\infty, \alpha$ be $\in [0;1]$. Then there exists $N \in \mathbb{N} \cup \{0\}$ such that $x^1, x^2 \in \mathcal{Y}(N)$, i.e. there exist $u^1(0), u^1(1), \ldots, u^1(N-1), u^2(0), u^2(1), \ldots, u^2(N-1) \in \mathcal{U}$ such that

$x^1(N) = x^1$, $x^2(N) = x^2$. According to (1) the formula is true for $x(0) = 0$:

$$\begin{cases} x^1 = x^1(N) = A^{N-1}u^1(0) + A^{N-2}u^1(1) + \ldots + u^1(N-1), \\ x^2 = x^2(N) = A^{N-1}u^2(0) + A^{N-2}u^2(1) + \ldots + u^2(N-1), \end{cases}$$

$$\begin{cases} \alpha x^1 = \sum\limits_{k=0}^{N-1} \alpha A^k u^1(N-k-1), \\ (1-\alpha)x^2 = \sum\limits_{k=0}^{N-1} (1-\alpha)A^k u^2(N-k-1), \end{cases}$$

$$(\alpha x_1 + (1-\alpha)x_2) = \sum_{k=0}^{N-1} A^k(\alpha u^1(N-k-1) + (1-\alpha)u^2(N-k-1)).$$

Due to the convexity of $\mathcal{U}$ the inclusion $\alpha u^1(N-k-1) + (1-\alpha)u^2(N-k-1) \in \mathcal{U}$, $k = \overline{0, N-1}$ is true. Then $\alpha x_1 + (1-\alpha)x_2 \in \mathcal{Y}(N) \subset \mathcal{Y}_\infty$. Which means that $\mathcal{Y}_\infty$ is convex.

□

Theorem 1 defines the basic apparatus for working with sets 2. Since $\mathcal{Y}_\infty$ is convex, various means of convex analysis can be used for its numerical modeling, for example, the method of polyhedral approximations. The following lemma is valid, which defines the structure of the reachable sets of the system $(A, \mathcal{U})$.

Lemma 1. [1, lemma 1] *For each $N \in \mathbb{N}$ the reachable set* (2) *of the system* $(A, \mathcal{U})$ *satisfies the relations:*

$$\mathcal{Y}(N) = \sum_{k=0}^{N-1} A^k \mathcal{U}, \ \mathcal{Y}(N) = A\mathcal{Y}(N-1) + \mathcal{U}.$$

Taking into account this representation, the reachable set coincides with the 0-controllability set up to the replacement of the matrix A by A^{-1}, the set $\mathcal{U}$ by $A^{-1}\mathcal{U}$. Then it follows from [11, lemma 4] that reachable sets will be bounded if and only if all eigenvalues of the matrix A modulo are strictly less than 1.

We denote the set of compacts in $\mathbb{R}^n$ by $\mathbb{K}_n$, and the Hausdorff distance [16] by ρ_H:

$$\mathbb{K}_n = \{\mathcal{X} \subset \mathbb{R}^n \colon \mathcal{X} - \text{compact}\},$$

$$\rho_H(\mathcal{X}, \mathcal{Y}) = \max\left\{ \sup_{x \in \mathcal{X}} \inf_{y \in \mathcal{Y}} \|x - y\|_p; \sup_{y \in \mathcal{Y}} \inf_{x \in \mathcal{X}} \|x - y\|_p \right\},$$

$$\|x\|_p = \sqrt[p]{\sum_{k=1}^{n} |x_k|^p}, \ p \geqslant 1.$$

If we consider that $\mathcal{U}$ is a convex compact in $\mathbb{R}^n$, then every set of the form (2) is also a convex compact, since it is representable as an algebraic sum of linear transformations of [14] compacts. Then in the metric space $(\mathbb{K}_n, \rho_H)$ you can define a mapping $T \colon \mathbb{K}_n \to \mathbb{K}_n$ of the following form:

$$T(\mathcal{Y}) = A\mathcal{Y} + \mathcal{U}. \tag{4}$$

Taking into account the lemma 1 and the relation (4), if the mapping T or $\underbrace{T \circ \ldots \circ T}_{M}$ for some $M \in \mathbb{N}$ are compressive, the limit of the sequence of reachable sets (2) in space $(\mathbb{K}_n, \rho_H)$ can be defined by the principle of contraction mappings [15]. Also, the principle of contraction mappings makes it possible to estimate the error of approximating the limit point using the simple iteration method. On the other hand, the limit point up to the closure by virtue of (3) must match with $\mathcal{Y}_\infty$. Let's formulate this fact in the form of a theorem.

Theorem 2. *Let all eigenvalues of the matrix $A \in \mathbb{R}^{n\times n}$ be strictly less than 1 modulo, the family $\{\mathcal{Y}(N)\}_{N=0}^{\infty}$ is defined by the ratio* (2)*, the set $\mathcal{Y}_\infty$ is defined by the ratio* (3)*, the mapping T has the form* (4)*.*

Then

1) there exists $M \in \mathbb{N}$ such that the mapping $T_M = \underbrace{T \circ \ldots \circ T}_{M}$ is compressive with some compression ratio $\alpha \in [0;1)$;
2) $\overline{\mathcal{Y}_\infty}$ is the only fixed point of the T in $(\mathbb{K}_n, \rho_H)$;
3) the estimation is true

$$\rho_H(\overline{\mathcal{Y}_\infty}, \mathcal{Y}(NM)) \leqslant \frac{\alpha^N}{1-\alpha}\rho_H(\mathcal{Y}(M), \{0\}).$$

Proof. The proof follows from [11, theorem 2] when replacing A^{-1} by A and $(-A^{-1}\mathcal{U})$ by $\mathcal{U}$.

□

The value of the compression coefficient α from the theorem 2 generally depends on the choice of the norm in the space $\mathbb{R}^n$ and, as a result, on the associated operator norm of the matrix A. For instance, the following estimates of the value of α are known when choosing different norms $\|\cdot\|_p$ in $\mathbb{R}^n$ [15]:

$$\alpha_1 = \max_{1\leqslant j\leqslant n}\sum_{i=1}^{n}|a_{ij}|,\ \ \alpha_2 = \sqrt{\sum_{i=1}^{n}\sum_{j=1}^{n}a_{ij}^2};\ \ \alpha_\infty = \max_{1\leqslant i\leqslant n}\sum_{j=1}^{n}|a_{ij}|. \qquad (5)$$

The methods that allow in the general case to determine at what value $M \in \mathbb{N}\cup\{0\}$ the T_M mapping will be compressive are currently unknown. However, taking into account the estimates of (5), the value of M can be determined numerically by sequentially calculating α for various values of $M \in \mathbb{N}\cup\{0\}$. Also, the choice of the norm in the space $\mathbb{R}^n$ affects the value of the Hausdorff distance in $\mathbb{K}_n$, which finally determines the structure of the external estimates of the set $\mathcal{Y}_\infty$.

Theorem 3. *Let all eigenvalues of the matrix $A \in \mathbb{R}^{n\times n}$ be strictly less than 1 modulo, the family $\{\mathcal{Y}(N)\}_{N=0}^{\infty}$ is determined by the relations* (2)*, the set $\mathcal{Y}_\infty$ is determined by the relation* (3)*, the value $M \in \mathbb{N}$ is chosen so that T_M is a contraction mapping with compression ratios $\alpha_1, \alpha_2, \alpha_\infty \in [0;1)$, which are associated with the norms $\|\cdot\|_1$, $\|\cdot\|_2$, $\|\cdot\|_\infty$ in the space $\mathbb{R}^n$ respectively. Then*

$$\mathcal{Y}_\infty \subset \mathcal{Y}(NM) + \operatorname{conv}\left\{(\underbrace{0,\ldots,0}_{i}, r, 0,\ldots,0)^T\colon r \in \{-R_1, R_1\},\ i = \overline{0, n-1}\right\},$$

$$\mathcal{Y}_\infty \subset \mathcal{Y}(NM) + \left\{x \in \mathbb{R}^n\colon \sqrt{\sum_{i=1}^{n}|x_i|^2} \leqslant R_2\right\},$$

$$\mathcal{Y}_\infty \subset \mathcal{Y}(NM) + \left\{x \in \mathbb{R}^n \colon \max_{i=\overline{1,n}} |x_i| \leqslant R_\infty \right\},$$

$$R_p = \frac{\alpha_p^N}{1-\alpha_p} \max_{x \in \mathcal{Y}(M)} \|x\|_p, \ p \in \{1,2,\infty\}, \ N \in \mathbb{N}.$$

Proof. By virtue of clause 3 of the theorem 2

$$\rho_H\left(\overline{\mathcal{Y}_\infty}, \mathcal{Y}(NM)\right) \leqslant \frac{\alpha_p^N}{1-\alpha_p^N} \rho_H(\mathcal{Y}(M), \{0\}) = R_p, \ p \in \{1,2,\infty\}.$$

Then by virtue of the definition of the Hausdorff distance

$$\mathcal{Y}_\infty \subset \overline{\mathcal{Y}_\infty} \subset \mathcal{Y}(NM) + B_{R_p}(0),$$

where

$$B_{R_1}(0) = \operatorname{conv}\left\{(\underbrace{0,\ldots,0}_{i}, r, 0, \ldots, 0)^T \colon r \in \{-R_1, R_1\}, \ i = \overline{0, n-1}\right\},$$

$$B_{R_2}(0) = \left\{x \in \mathbb{R}^n \colon \sqrt{\sum_{i=1}^n |x_i|^2} \leqslant R_2 \right\},$$

$$B_{R_\infty}(0) = \left\{x \in \mathbb{R}^n \colon \max_{i=\overline{1,n}} |x_i| \leqslant R_\infty \right\}.$$

□

The theorem 3 allows you to construct external estimates of the set $\mathcal{Y}_\infty$ with any predetermined accuracy.

We demonstrate the theoretical results obtained by using the example of constructing a limit reachable set for a linear discrete-time system of the form (1).

Example 1. We consider the three-dimensional system $(A, \mathcal{U})$, where

$$A = \begin{pmatrix} 0{,}25 & 0 & 1 \\ 0 & -0{,}16 & 0 \\ 0 & 0 & 0{,}14 \end{pmatrix},$$

$$\mathcal{U} = \operatorname{conv}\left\{ \begin{pmatrix} 4 \\ 4 \\ 3 \end{pmatrix}, \begin{pmatrix} 2 \\ 4 \\ -3 \end{pmatrix}, \begin{pmatrix} -2 \\ 2 \\ 0 \end{pmatrix}, \begin{pmatrix} -4 \\ -4 \\ 3 \end{pmatrix}, \begin{pmatrix} -2 \\ -4 \\ 3 \end{pmatrix}, \begin{pmatrix} 2 \\ -2 \\ 0 \end{pmatrix}, \begin{pmatrix} 0 \\ 0 \\ 4 \end{pmatrix}, \begin{pmatrix} 0 \\ 0 \\ -4 \end{pmatrix} \right\}.$$

Let us construct for the system $(A, \mathcal{U})$ an estimate of the limit reachable set $\mathcal{Y}_\infty$ according to the theorem 3. As the value of the parameter defining the norm in $\mathbb{R}^3$, we choose $p = 1$. Then

$$\|A\| = \alpha_1 = 0{,}25, \ M = 1, \ \max_{x \in \mathcal{Y}(M)} \|x\|_1 = 11, R_1(N) = \frac{(0{,}25)^N}{1 - 0{,}25} 11.$$

We construct an external estimate $\mathcal{Y}_\infty$ for $N = 3$:

$$\hat{\mathcal{Y}}_\infty = \mathcal{Y}(3) + \left\{x \in \mathbb{R}^3 : |x_1| + |x_2| + |x_3| \leqslant R_1(3)\right\}.$$

The estimate $\hat{\mathcal{Y}}_\infty$ for $N = 3$ is shown in the Fig. 1.

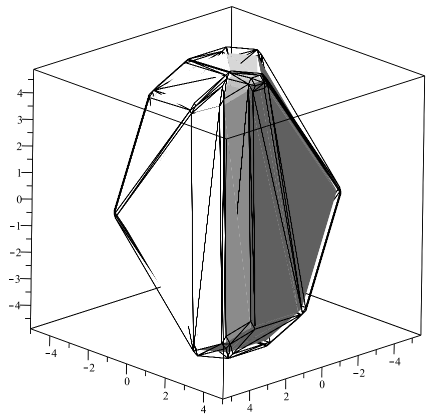

Fig. 1. The reachable set $\mathcal{Y}(3)$ is denoted by a polyhedron, the solid line denotes the external estimate of the limit reachable set for $p = 1$

3. Contructing Limit Reachable Sets for the Damping System of a High-rise Structure

As a demonstration of the effectiveness of the method developed in section 2, we propose a mathematical model of a high-rise structure located in a zone of seismic activity, and for the selected control system we construct limit reachable sets. As a mechanical system modeling the vibrations of a high-rise structure, a one-dimensional sequence of elastically connected material points (floors or sections of the structure) is assumed, one of which (the base) makes a translational motion generated by seismic action.

We take an assumption that the mass of the base is much higher than the masses of other material points, therefore, the influence of the movement of sections of the structure on the movement of the base can be ignored. In the future we will assume that the masses of all material points are the same, and elastic and damping bonds are modeled by linear elements with the same coefficients of elasticity and damping.

The equations of motion of the system under consideration, according to the model proposed in [17], have the form

$$\begin{cases} m\ddot{\xi}_1(t) &= -\,2b\dot{\xi}_1(t) - 2c\xi_1(t) + b\dot{\xi}_2(t) + c\xi_2(t) + U_1(t), \\ \vdots & \\ m\ddot{\xi}_i(t) &= -\,2b\dot{\xi}_i(t) - 2c\xi_i(t) + b\dot{\xi}_{i-1}(t) + \\ & \qquad + c\xi_{i-1}(t) + b\dot{\xi}_{i+1}(t) + c\xi_{i+1}(t) + U_i(t), \\ \vdots & \\ m\ddot{\xi}_n(t) &= -\,2b\dot{\xi}_n(t) - 2c\xi_n(t) + b\dot{\xi}_{n-1}(t) + c\xi_{n-1}(t) + U_n(t), \end{cases} \tag{6}$$

where ξ_i is the coordinate of the ith material point relative to the base, U_i is the control force applied to the ith material point, m is the mass of the material point, b and c are the damping and elasticity coefficients of intersectional bonds, respectively.

We assume that the control is relay-based, i.e. $U(t) = v_k$, $t \in [k\Delta t; (k+1)\Delta t)$ for an arbitrary $k \in \mathbb{N}$ and a fixed sampling step $\Delta t > 0$. Then we can proceed to the following equivalent relations:

$$\begin{aligned} &z(k+1) = A_d z(k) + \tilde{u}(k), \\ &z(0) = y_0,\ \tilde{u}(k) \in B\mathcal{U}_0, \end{aligned} \tag{7}$$

where $A_1 \in \mathbb{R}^{2n\times 2n}$ is the matrix of the discretized system, $B\mathcal{U}_0 \subset \mathbb{R}^{2n}$ is the set of admissible control values, $z(k) = (\xi(k\Delta t)^T, \dot{\xi}(k\Delta t))^T \in \mathbb{R}^{2n}$ is the state of the system at the kth step, $\Phi(t)$ is the matrix of the fundamental decision system (6). The following notations are used:

$$\begin{aligned} &A_d = \Phi(\Delta t)\Phi^{-1}(0), \\ &B = \Phi(\Delta t)\Phi^{-1}(0)\tilde{A}^{-1} - \tilde{A}^{-1}, \\ &\tilde{u}(k) = Bv_k,\ k \in \mathbb{N} \cup \{0\}. \end{aligned}$$

We assume that control installations on different floors operate independently of each other, and only restrictions related to the maximum power of control actions are imposed on relay control modes $u_{\max} > 0$,

$$\mathcal{U}_0 = \{0\}^{10} \times [-u_{\max}; u_{\max}]^{10}.$$

For the (7) system, the condition $0 \in \operatorname{int} B\mathcal{U}_0$, necessary for constructing limit reachable sets, is not fulfilled, since the dimension of this set of admissible control values does not coincide with the dimension of the phase space. For this reason, let us move on to an auxiliary system of the form (1), doubling the quantization step:

$$\begin{aligned} &A = A_d^2,\ \mathcal{U} = A_d B\mathcal{U}_0 + B\mathcal{U}_0, \\ &x(k) = z(2k),\ u(k) = A_d B\tilde{u}(2k) + B\tilde{u}(2k+1),\ k \in \mathbb{N} \cup \{0\}. \end{aligned} \tag{8}$$

Taking into account the definition of (3), the limit reachable sets of the (7) and (1) systems coincide under the assumptions of (8).

Let the height of the building be 10 floors, i.e. $n = 10$. Also let $\Delta t = 1$, $m = 600{,}000$, $b = 600{,}000$, $c = 2{,}400{,}000$. The parameter values are selected based on the model described in [17]. We accept $u_{\max} = 1$, because by virtue of Lemma 1 and definition (3), the limit reachable sets corresponding to different values of $u_{\max}$ will be proportional to each other.

The matrix A has 10 pairs of complex conjugate eigenvalues:

$$\begin{gathered} \lambda_{1,2} = 0{,}388 \pm 0{,}836I,\ \lambda_{3,4} = -0{,}446 \pm 0{,}574I, \\ \lambda_{5,6} = -0{,}498 \pm 0{,}05I,\ \lambda_{7,8} = -0{,}161 \pm 0{,}265I,\ \lambda_{9,10} = 0{,}042 \pm 0{,}174I, \\ \lambda_{11,12} = 0{,}078 \pm 0{,}064I,\ \lambda_{13,14} = 0{,}058 \pm 0{,}01I, \\ \lambda_{15,16} = 0{,}036 \pm 0{,}007I,\ \lambda_{17,18} = 0{,}022 \pm 0{,}01I,\ \lambda_{19,20} = 0{,}016 \pm 0{,}011I. \end{gathered}$$

The matrix A can be reduced to its real Jordan form $\Lambda \in \mathbb{R}^{20\times 20}$, which consists of cells $A_i \in \mathbb{R}^{2\times 2}$, $i = \overline{1,10}$, by means of a non-degenerate linear transformation $S \in \mathbb{R}^{20\times 20}$.

Since all eigenvalues modulo are strictly less than 1, by virtue of [11, lemma 4] the limit reachable set $\mathcal{Y}_\infty \subset \mathbb{R}^{20}$ of the (8) system is bounded and can be estimated from above as follows: $\mathcal{Y}_\infty \subset S\left(\mathcal{Y}_{1,\infty} \times \ldots \times \mathcal{Y}_{10,\infty}\right)$, where $\mathcal{Y}_{i,\infty}$ is the limit reachable set of the subsystem $(A_i, P_i S^{-1}\mathcal{U})$, $i = \overline{1,10}$. Here $P_i \in \mathbb{R}^{2\times 20}$ denotes the projection matrix on the plane corresponding to $(2i-1)$th and $(2i)$th coordinates: $P_i = (\underbrace{O\ \ldots\ O}_{i-1}\ I\ O\ \ldots\ O)$.

To estimate the sets $\mathcal{Y}_{i,\infty}$, $i = \overline{1,10}$, we use the theorem 3. As a demonstration, we consider $i = 1$ and perform calculations similar to Example 1. Fig. 2 shows the calculation results for the following numerical values of the parameters $p = 1$ and $p = \infty$.

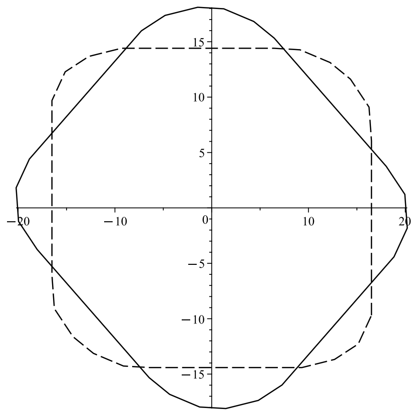

Fig. 2. The external estimate of the limit reachable set for $p = 1$ is denoted by a solid line, the external estimate for $p = \infty$ is denoted by a dashed line

Conclusion

We developed the method in the paper for numerical modeling of the limit reachable sets of linear discrete-time systems with bounded control. The set of admissible control values is assumed to be a convex compact containing the origin. For the case of a limited limit reachable set, a method is proposed for constructing its external estimate based on the principle of contraction mappings with any predetermined accuracy in the shape of a polyhedron, which allows the modeling process to be implemented numerically.

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О МЕТОДЕ ЧИСЛЕННОГО МОДЕЛИРОВАНИЯ ПРЕДЕЛЬНЫХ МНОЖЕСТВ ДОСТИЖИМОСТИ ДЛЯ ЛИНЕЙНЫХ ДИСКРЕТНЫХ СИСТЕМ С ОГРАНИЧЕННЫМ УПРАВЛЕНИЕМ

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В данной работе разработан метод для линейных дискретных систем, который основан на принципе сжимающих отображений. Этот метод предназначен для построения внешней оценки предельного множества достижимости, что является частой задачей в теории управления и анализе динамических систем. Применение принципа сжимающих отображений позволяет обеспечить возможность получения оценки с произвольным порядком точности в смысле расстояния Хаусдорфа. С другой стороны, предельная точка с точностью до замыкания должна совпасть с предельным множеством достижимости. Значение коэффициента сжатия зависит от выбора нормы в пространстве векторов, что, соответственно, влияет на значение расстояния Хаусдорфа в пространстве компактов и операторную норму матрицы системы. Для демонстрации возможностей предложенного метода представлен пример трехмерной системы с действительными собственными значениями. Также представлен пример применения метода для задачи построения предельного множества достижимости в системе демпфирования высотного сооружения в зоне сейсмической активности.

Ключевые слова: дискретная система; предельное множество достижимости; принцип сжимающих отображений; выпуклое множество; полиэдральная аппроксимация.

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