MSC 00A71 DOI: 10.14529/mmp240403

A MODEL AND A NUMERICAL METHOD FOR OPTIMIZING THE CHOICE OF A TRAINING TRAJECTORY FOR HETEROGENEOUS GROUPS OF SPECIALISTS

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Training and retraining of specialists of different profiles at present requires taking into account the high dynamics of the conditions of their professional activity. This is especially relevant when it is necessary to train specialists to act in emergency situation. For this reason, two major problems with organisation of the training process of specialists have arisen:

- the requirement for simultaneous training of a heterogeneous group which consists of specialists of different profiles who jointly provides the solution of a certain range of tasks in case of emergencies;
- the requirement for minimizing the duration of the training process.

Both universal and individual competences are expected of specialists in heterogeneous groups. In particular, in heterogeneous groups that prepare for emergency response, universal competences are required to act in special circumstances and individual competences are required to fulfil narrow professional tasks.

The said circumstance makes it possible to organize the sequence of courses for training specialists in the groups under consideration in such a way that it is possible to obtain universal competences in one course simultaneously by specialists of different profiles, which allows reducing the total training time of the whole heterogeneous group. At the same time, it is necessary to take into account the capabilities of the educational organisation in terms of the number of simultaneous trainees in each course, which ensures the acquisition of the relevant competence.

In this regard, there is a need to optimize the choice of trajectory, i.e., the sequence of courses, for training specialists in heterogeneous groups, taking into account the capacity of the educational organisation that trained them. For this purpose, we have developed a mathematical model and a numerical method for finding the optimal trajectory based on the use of genetic algorithm, the advantage of which is polynomial computational complexity. A numerical example is presented.

Keywords: specialist training; heterogeneous groups; learning trajectory optimization; genetic algorithm.

Introduction

In today's situation, the need to train groups of specialists of different profiles in the shortest possible time in order for them to jointly solve a certain range of tasks is becoming more and more important [1, 2]. Examples are medical brigades and specialised rescue groups designed to operate in an emergency zone of a unique nature.

The duration of training of groups of specialists acts as the main criterion of training efficiency. Note that some of the competences obtained during the training of specialists may be the same, while the remaining competences reflect their specialisation [3, 4]. The organisation of the training process, which combines the process of obtaining common competences by specialists of different profiles, allows to reduce the training time for all specialists of the group. The resources of the training organisation should be taken into account.

In this regard, there is a need to optimise the choice of training trajectory of specialists, which will minimise preparation time of the whole group taking into account the available resources. The solution to this problem can be obtained using mathematical modelling methods, which is the subject of this paper.

1. Formalisation of the Process of Obtaining Competencies in the Implementation of Specialist Training Programmes

The educational trajectory of a group of specialists is defined as the composition and sequence of individual courses undertaken by each specialist [5]. Owing to the limitations of the educational organisation, no more than a given number of individuals can participate in each course simultaneously. Situations in which the trajectory includes time gaps between the courses are not excluded. In such cases, it is assumed that these gaps are filled by other forms of training (e.g., independent study), which fall outside the scope of this study. Consequently, specifying the educational trajectory of a group of specialists must also include the start time for each course for each specialist, i.e., course scheduling [6, 7].

Information about the capacity of the educational organisation is necessary for determining the composition of the courses. Assume that the training of a heterogeneous group of specialists necessitates the acquisition of a set of competencies, each of which is attained through completion of one specific course from the set of available courses $U = \{u_1, ..., u_{|U|}\}$.

Let us denote τ_k is a training duration on course u_k, θ is the unit of measurement for the duration of all courses is such that $\forall k\ \tau_k \vdots \theta$, $T = \{0, 2\theta, 3\theta, ...\}$ is a discrete set of potential moments of start or end of training by courses. For each course u_k binary tuple is defined $\kappa_k^{in} = \left(\kappa_{k1}^{in}, \ldots, \kappa_{kQ}^{in}\right)$ course features, required to start training on it:

$$\kappa_{ks}^{in} = \begin{cases} 1, & \text{if the course } u_k \text{ is required to start a course of study } u_s; \\ 0, & \text{otherwise;} \end{cases}$$

and a binary tuple $\kappa_k^{out} = \left(\kappa_{k1}^{out}, \ldots, \kappa_{k|U|}^{out}\right)$, contains a characterisation of the course of study u_k

$$\kappa_{ks}^{out} = \begin{cases} 1, & \text{if } k = s; \\ 0, & \text{otherwise.} \end{cases}$$

The training of the i-th specialist assumes that he passes a given set of courses. Let us denote $\nu_i = \left(\nu_{i1}, \ldots, \nu_{i|U|}\right)$ is a tuple of characterisation of the courses, completion of which is necessary for the i-th specialist, where

$$\nu_{ik} = \begin{cases} 1, & \text{if the course } u_k \text{ is required for the } i\text{-th specialist;} \\ 0, & \text{otherwise.} \end{cases}$$

Note that there is a partial ordering of courses and therefore there may be alternative versions of the sequence of courses required for specialists. The sequence of courses in the p-th variant for the i-th specialist is described as

$$\left(u_{p_i 1}, \ldots, u_{p_i w_i}\right), \tag{1}$$

where $w_i = \sum\limits_{k=1}^{|U|} \nu_{ik}$ is the number of courses required for the training of i-th specialist.

All courses included sequences of the form (1), consist of different elements of the set U and they can be identified with the courses of the educational organisation using the following variables:

$$\nu_{ik} = \begin{cases} 1, & \text{if } u_{p_il} = u_k \in U; \\ 0, & \text{otherwise.} \end{cases}$$

In this case $\kappa^{in}_{p_ils} = f_{p_ilk}\kappa^{in}_{ks}$, $s = 1, ..., |U|$, where $\kappa^{in}_{p_il} = \left(\kappa^{in}_{p_il1}, \ldots, \kappa^{in}_{p_il|U|}\right)$, $l = 1, ..., w_i$, $\kappa^{out}_{p_ils} = f_{p_ilk}\kappa^{out}_{ks}$, $s = 1, ..., |U|$, $\kappa^{out}_{p_il} = \left(\kappa^{out}_{p_il1}, \ldots, \kappa^{out}_{p_il|S|}\right)$, $l = 1, ..., w_i$.

Correctness of the sequence (1) of courses for each variable is determined by the following relations: no prior learning of competences is required for the first course of study:

$$\forall i \forall p \kappa^{in}_{p_i1} = (0, ..., 0); \tag{2}$$

all the required competences must be obtained in the subsequent courses

$$\forall i \forall p \ \kappa^{in}_{p_is+1} \leq \sum_{l=1}^{s} \kappa^{out}_{p_il}; \, s = 1, ..., w_i - 1; \tag{3}$$

training in all courses for each option should ensure that all the necessary competencies for specialists in the given field are obtained:

$$\forall i \forall p \sum_{l=1}^{w_i} \kappa^{out}_{p_il} \geq \nu_i. \tag{4}$$

2. Model of Optimization of the Process of Competence Acquisition in the Implementation of Specialist Training Programmes

Let us denote N is the number of specialists, g_k is the maximum number of professionals who can be trained in the course $u_k \in U$ at the same time. Let's enter variables

$$x_{tp_il} = \begin{cases} t, & \text{if at the } t\text{-th moment of time } i\text{-th specialist is trained by the} \\ & p\text{-th variant and starts studying the course } u_{p_il}; \\ 0, & \text{otherwise.} \end{cases}$$

Note that the conditions

$$\forall t \, \forall k \sum_{i=1}^{N} \sum_{p=1}^{w_i} \operatorname{sgn}\bigl(x_{tp_il}\bigr) \cdot f_{p_ilk} \leq g_k \tag{5}$$

describe the educational organisation's restriction on the number of specialists simultaneously enrolled in each course.

Since the variables x_{tp_il} and expressions $x_{tp_il} + f_{p_ilk} \cdot \tau_k$ completely describe the moments of beginning and ending of training of all specialists, the set of all variables $X = \{x_{tp_il}\}$ completely characterises the trajectory of group training of all specialists.

In this case, the problem of optimising the choice of the trajectory of group training of a group of specialists has the form of finding

$$X^* = \operatorname{Arg\,min} \max_{i,p,l} \left(x_{tp_il} + f_{p_ilk} \cdot \tau_k\right), \tag{6}$$

under restrictions (2) – (5). Model (2) – (6) is a nonlinear mathematical programming problem. To develop a numerical method for its solution, the genetic algorithm can be used, the advantage of which is polynomial complexity $O(n^3)$ [8, 9]. The following is a description of the variant of this algorithm for the problem under consideration.

3. Description of the Numerical Method

To describe a genetic algorithm, the definitions of genes, chromosomes and individuals must initially be given. As follows from the problem statement, for each specialist there is a set of competences to be acquired in the course of training. By virtue of the assumption described above, a specific training course from the given set of courses U is defined for obtaining each competence. Consequently, a priori for each specialist, there are many courses that he or she must take during the training period. Therefore, it is reasonable to take as a gene a separate course studied by a specialist, and as a chromosome a sequence (1) of courses studied, which does not contradict the causal relations between the processes κ_k^{in} of competence acquisition described by tuples. In this case, the individual is a set of chromosomes corresponding to all the specialists in training.

Let us turn to the description of the adaptation function of individuals. As stated above, the capacity of the educational organisation has limitations on the number of simultaneous students on each course, which leads to the need for scheduling of courses by specialists [7]. The minimization of the total time for all specialists to complete all courses is used as an optimization criterion in accordance with the problem formulation [10, 11].

Hence, it is possible to design an optimal schedule of courses to be taken by specialists and the length of the optimal schedule can be taken as an estimate of the fitness of an individual.

Thus, the values of the fitness function of individuals are determined by the type of chromosomes, i.e., the sequences (1) of learning courses and, by choosing such sequences for specialists, it is possible to achieve an improvement in the fitness function. Changing the selection of chromosomes corresponding to one specialist can be done in two ways: crossbreeding and mutation.

Let's move on to a description of these methods. We will crossbreed separately for each pair of chromosomes of each individual corresponding to the same specialist. Let us denote n is the number of courses for a specialist. Let us assume that the crossed chromosomes are represented by tuples $A = (a_1, ..., a_n)$ and $B = (b_1, ..., b_n)$; the result of crossbreeding is a tuple $C = (c_1, ..., c_n)$. If $a_1 = b_1$, then $c_1 - a_1 = b_1$, otherwise $c_1 = a_1 \oplus b_1$, where $\oplus$ is a “exclusive-or” operation, an outcome that is determined randomly.

Then, until all elements of a tuple C have been determined, the elements of the tuples A and B compared sequentially, so that

1) exclude the possibility of double occurrence of the same gene (course of study);

2) to exclude the possibility of violation of cause-and-effect relationships in the study of courses;

3) the following actions are performed to select the next element c_k, $k = 2, ..., n$ of the course tuple C:

- locate a_i is the first unexamined element of the tuple A, that has not been included in the tuple C before;
- locate b_i is the first unexamined element of the tuple B , that has not been included in the tuple C before;

- the existence of a cause-and-effect relationship $\prec$ between the courses a_i and b_i, specified by the corresponding tuple is checked κ^{in}:

– if $a_i \prec b_j$, i.e. $\kappa^{in}_{ij} = 1$, then $c_k = a_i$; – if $b_j \prec a_i$, i.e. $\kappa^{in}_{ji} = 1$, then $c_k = b_j$; – if both conditions are not fulfilled, i.e. $\kappa^{in}_{ij} = \kappa^{in}_{ji} = 0$, then $c_k = a_i \oplus b_j$.

The block diagram of the algorithm of crossover functioning at crossing of two chromosomes is presented in Fig. 1.

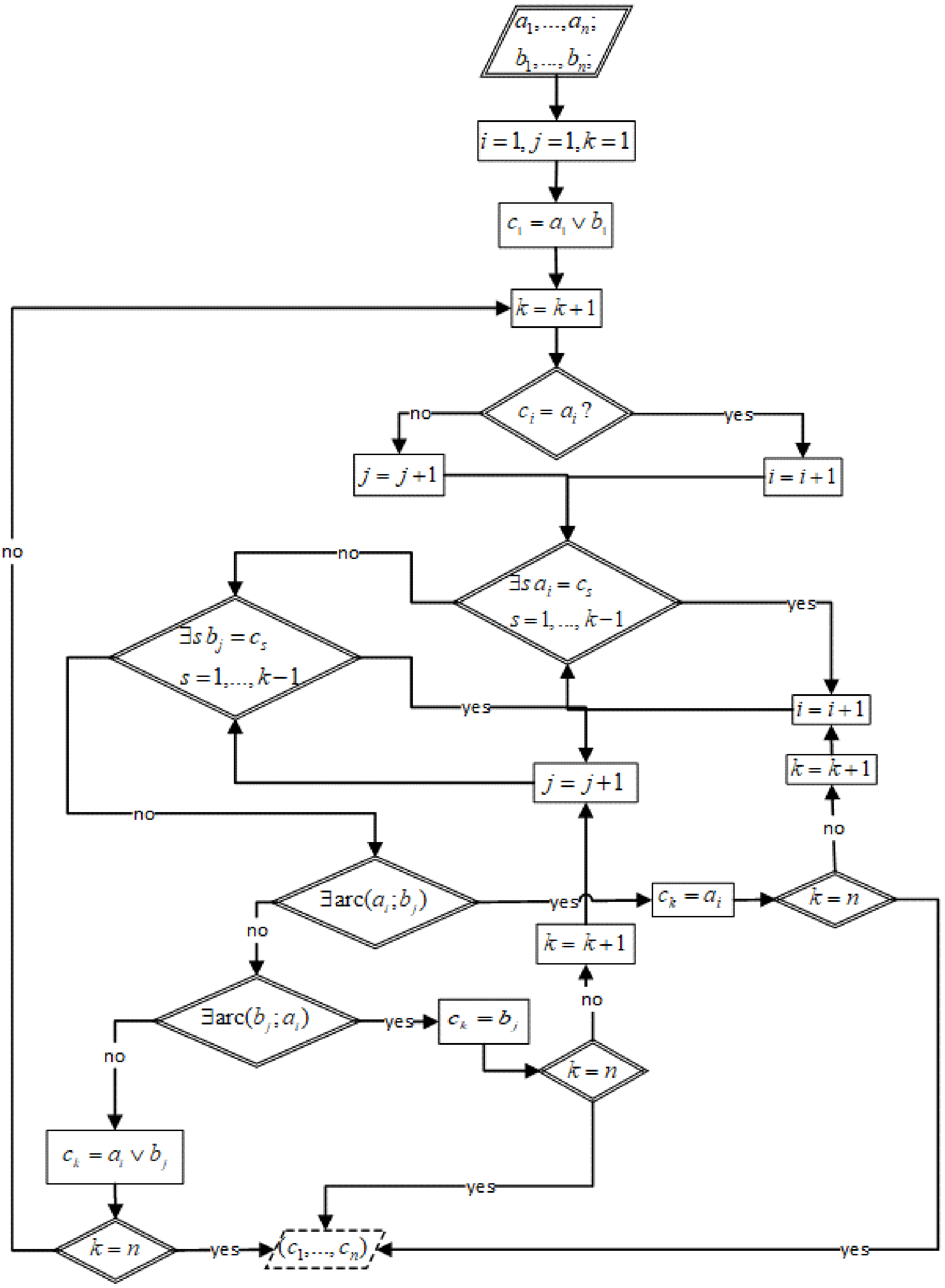


Fig. 1. Block diagram of the algorithm of crossbreeding of individuals

Let's describe the process of mutation of an individual. Each mutation must be carried out for randomly selected chromosomes, independently of other chromosomes [8]. This is because the composition of courses (competences) for each specialist is strictly defined and cannot be changed. Mutations are permissible rearrangements of genes in the chromosomes of an individual and occur in the following sequence. Chromosomes are randomly selected to carry out the mutation (from 0 to the number of specialists).

For each selected chromosome, the following are randomly determined:

1) number of genes to mutate;
2) locations of mutated genes;
3) sequence of mutated gene rearrangements.

For each location of the mutated gene, the following are randomly determined:

1) direction of permutation (right, left);
2) number of permutations.

For each mutated gene, a maximum permissible number of permutations is carried out, not exceeding the previously determined number, in the direction determined therein. The permissible number of permutations is determined so as not to disturb the cause-and-effect relations between the courses (genes).

Thus, the following values must be set for the mutation to occur in an individual: the probability of selecting chromosomes to make a mutation; probability of selecting genes for permutations in the mutated chromosome; directions and number of permutations for each gene.

Next, the mutation operation is described using the example of a chromosome $(u_3, u_2, u_6, u_5, u_1, u_8)$, in which it is necessary to move the gene u_5 to the left by 3 positions. After analysing the cause-effect relations between the courses, it turned out that u_5 cannot be moved to the left of u_2 and, therefore, the move is possible only by 1 position (Fig. 2).

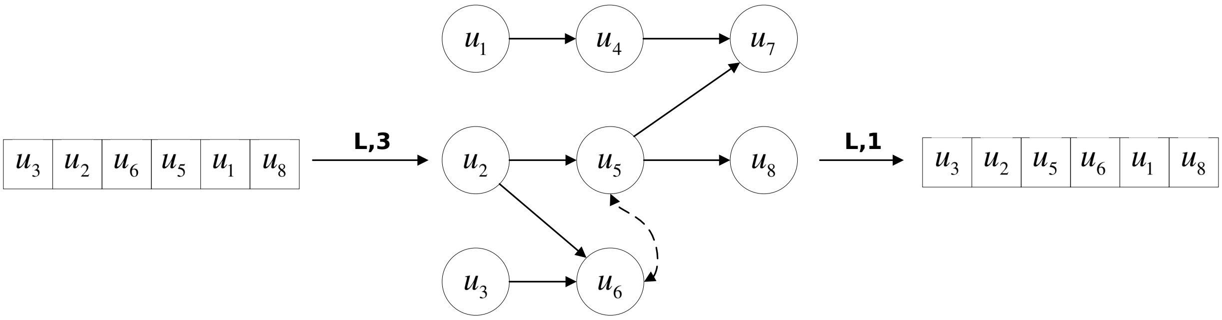


Fig. 2. Example of a single chromosome mutation

4. Numerical Example

Consider the process of training 6 specialists, where by three specialists must master the courses $\{u_1,\ u_2,\ u_3,\ u_5,\ u_6,\ u_8\}$, two courses $\{u_1,\ u_2,\ u_3,\ u_6\}$ and one courses $\{u_1,\ u_2,\ u_4,\ u_7\}$. Course durations are considered to be the same and are assumed to be equal to 1. The cause-and-effect relationships between courses are presented in Fig. 3, with an indication of the educational organisation's limitation on the number of concurrent specialists. After ten iterations of crossing, we obtain the following optimal schedule Fig. 4.

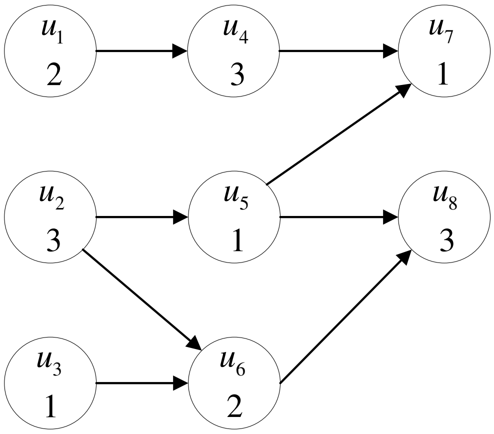
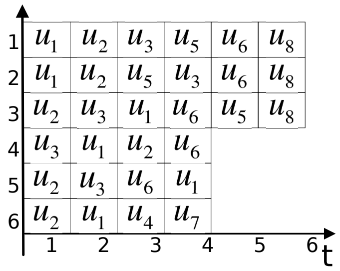


Fig. 3. Causal relationships between courses in an educational organisation



	1	2	3	4	5	6
1	u_1	u_2	u_3	u_5	u_6	u_8
2	u_1	u_2	u_5	u_3	u_6	u_8
3	u_2	u_3	u_1	u_6	u_5	u_8
4	u_3	u_1	u_2	u_6		
5	u_2	u_3	u_6	u_1		
6	u_2	u_1	u_4	u_7		

Fig. 4. Optimal timetable

Conclusion

The stopping criterion was a threefold subsequent repetition of the obtained result. The above-described model and numerical optimization method based on the genetic algorithm allow selecting a training trajectory for heterogeneous groups of specialists taking into account the organisation's resources. It is assumed that a priori sets of necessary new competences are known for each specialist of the heterogeneous group. This approach allows increasing the efficiency of the training process by taking into account the already known competences of each specialist.

At the same time, there is often a task of such selection of specialists into the group to be formed on the basis of their existing competences, which would allow to form a heterogeneous group of specialists possessing the given competences in a minimum of time [12]. The solution to this problem is the aim of further research and can be derived from the results of this paper.

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Received September 19, 2024

УДК 004.94+004.85 **DOI: 10.14529/mmp240403**

МОДЕЛЬ И ЧИСЛЕННЫЙ МЕТОД ОПТИМИЗАЦИИ ВЫБОРА ТРАЕКТОРИИ ПОДГОТОВКИ ГЕТЕРОГЕННЫХ ГРУПП СПЕЦИАЛИСТОВ

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Подготовка и переподготовка специалистов различных профилей в настоящее время требуют учёта высокой динамики условий их профессиональной деятельности. Особую актуальность это приобретает при необходимости подготовки специалистов к действиям в чрезвычайных обстоятельствах. В этом случае возникают две основные проблемы организации процесса обучения специалистов:

- необходимость одновременной подготовки гетерогенной группы, включающей специалистов различного профиля, обеспечивающих совместно решение определённого круга задач при возникновении чрезвычайных обстоятельств;
- необходимость минимизации длительности процесса подготовки.

В гетерогенных группах предполагается, что специалисты должны обладать как универсальными, так и индивидуальными компетенциями. В частности, в гетерогенных группах, которые готовятся для действий в чрезвычайных обстоятельствах, универсальные компетенции необходимы для действий в особых условиях, а индивидуальные – для выполнения узкопрофессиональных задач.

Указанное обстоятельство позволяет так организовать последовательность курсов для подготовки специалистов в рассматриваемых группах, чтобы универсальные компетенции было возможно получать на одном курсе одновременно специалистами различного профиля, что позволяет сокращать общее время подготовки всей гетерогенной группы. При этом необходимо учитывать возможности образовательной организации по количеству одновременно обучающихся на каждом курсе, обеспечивающем получение соответствующей компетенции.

В связи с этим возникает необходимость оптимизации выбора траектории, т. е. последовательности курсов, подготовки специалистов в гетерогенных группах с учётом возможности образовательной организации, осуществлявшей их подготовку. С этой целью была разработана математическая модель и численный метод нахождения оптимальной траектории, основанный на использовании генетического алгоритма, преимуществом которого является полиномиальная вычислительная сложность. Приведён численный пример.

Ключевые слова: подготовка специалистов; гетерогенные группы; оптимизация траектории обучения; генетический алгоритм.

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Поступила в редакцию 19 сентября 2024 г.