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NUMERICAL IDENTIFICATION OF HYDRODYNAMIC PARAMETERS OF A RESERVOIR UNDER ELASTIC-WATER-DRIVE DEVELOPMENT MODE

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The process of oil reservoir development in the elastic-water-drive mode is considered. It is assumed that the displacement of oil by the edge water occurs completely and a clear boundary between two liquids is formed in the reservoir, which moves according to a previously unknown law. Within the framework of a one-dimensional model of the elastic-water-drive development regime, the task is set to identify the main hydrodynamic parameters of the reservoir, i.e. the pressure at the interface between liquids, the pressure distribution in the reservoir and the position of the interface between liquids, only on the basis of information obtained from the gallery of production wells. The problem set belongs to the class of boundary inverse problems.

By applying the methods of front straightening and difference approximation, the problem is reduced to solving a system of difference equations. A special representation is proposed to solve the system of difference equations, having previously written it down as a variational problem with local regularization. As a result, an explicit formula is obtained for determining the approximate value of the pressure at the interface of liquids and recurrent formulas for determining the distribution of pressure and the position of the interface of liquids in the reservoir at each time layer. Based on the proposed computational algorithm, numerical experiments were carried out for a model oil reservoir.

Keywords: elastic-water-driven development mode; boundary inverse problem; front straightening method; local regularization; difference method.

Introduction

It is known that in oil fields with an initial reservoir pressure higher than the saturation pressure of oil with gas, at the initial stage, development is carried out in a natural elastic mode. Under the conditions of an elastic development mode, oil is in a single-phase state and the inflow of oil to the wells occurs due to the use of the potential energy of elastic deformation of the reservoir and oil [1]. And if the oil deposit is connected with the surrounding reservoir water-pressure system, then an elastic-water-pressure mode develops during development. When this mode manifests itself, the movement of oil to the wells occurs not only due to the potential energy of elastic deformation of the reservoir and oil, but also due to the energy of the pressure of the marginal waters in the aquifer area [2, 3]. Under the elastic-water-pressure mode, the development of the deposit stops when the marginal water reaches the oil wells and water is extracted from the reservoir instead of oil.

Experimental studies of processes occurring during development of oil fields in the elastic-water-drive mode show that in most cases the displacement of oil by the edge water occurs completely and a clear boundary between the two liquids is formed in the reservoir, moving according to a previously unknown law. At the same time, the dynamics of pressure distribution in the reservoir, the terms of operation, flooding, etc. depend on the rate of movement of the boundary between the liquids. And the dynamics of pressure distribution, in turn, affects almost all indicators of reservoir development, including well productivity and change in the development mode. Therefore, for the practice of developing oil fields with an elastic-water-drive mode, the issue of monitoring the dynamics of the main hydrodynamic parameters of the reservoir, including the pressure at the boundary between the liquids, the pressure distribution in the reservoir and the position of the boundary between the liquids, is very important. This issue is also relevant for the processes of developing gas fields with active edge water, the creation and operation of underground gas storage facilities in aquifers and depleted flooded fields.

It should be noted that wells penetrating the reservoir in fairly small areas are the main sources of information about the processes occurring in the reservoir during development. Access to the oil reservoir is possible only through wells where the pressure and oil flow rate (well flow rate) are directly measurable. However, the position of the liquid interface and the pressure at any point in the reservoir (except for well locations) are not directly measurable. Consequently, it is not possible to control the dynamics of pressure distribution and the movement of the liquid interface by directly measuring these reservoir parameters. In this regard, in order to control the dynamics of pressure distribution and the movement of the liquid interface, it is necessary to use a mathematical model of the elastic-water-drive regime of reservoir development [2–4]. In this paper, the problem of identifying the main hydrodynamic parameters of the reservoir is presented as a boundary inverse problem for a one-dimensional model of the elastic-water-drive regime of development.

1. Statement of the Problem

Let us consider a horizontally located rectangular oil-bearing layer, with an initial pressure higher than the saturation pressure of oil by gas, with a length l , with a constant width r and thickness h. The layer is limited from above and below by impermeable planes and is considered a homogeneous, weakly deformable porous medium. At the entrance boundary of the formation $x = 0$ there is a gallery of production wells, and the outer boundary of the formation $x = l$ is surrounded by edge water.

Let us assume that at the moment of time $t = 0$ a gallery of production wells is put into operation. Due to the potential energy of elastic deformation of the formation and oil, as well as due to the pressure of the marginal water, a rectilinear-parallel flow of oil to the gallery of wells occurs, i.e. an elastic-water-drive regime develops in the formation. As oil is withdrawn through the gallery, the marginal water enters the formation, completely replacing the pores occupied by oil, and a clear boundary between oil and water is formed. It is assumed that oil is a weakly compressible viscous liquid and its movements in the formation obey Darcy's law. Then the mathematical model of the elastic-water-drive regime of development of this formation can be presented in the form [1–4]

$$\frac{\partial P(x,t)}{\partial t} = \chi \frac{\partial^2 P(x,t)}{\partial x^2}, \ \ (x,t) \in \Omega_s = \{0 < x < s(t), \ 0 < t \leq T\}, \qquad (1)$$

where $P(x,t)$ is the pressure, $\chi = \frac{k}{\phi_0 (c_f + c_r)\mu}$ is the coefficient of pyroconduc-tivity of the formation, k is the coefficient of absolute permeability of the formation, μ is the dynamic viscosity of oil, c_f is the coefficient of oil compressi-bility, c_r is the coefficient of elasticity of the formation, ϕ_0 is the coefficient of porosity of the formation at a fixed pressure, $s(t)$ is the position of the water-oil interface. Let at the initial moment of time $t = 0$ the pressure distribution in the oil-bearing formation and the position of the liquid interface be known, i.e. for equation (1) we have the following initial conditions

$$P(x,0) = \psi(x), 0 \leq x \leq s(0), s(0) = l. \qquad (2)$$

Let us assume that the change in the flow rate of the production gallery over time is described by the function $q(t)$. Then at the input boundary of the formation $x = 0$ we will have the following condition

$$rh \frac{k}{\mu} \frac{\partial P(0,t)}{\partial x} = q(t). \qquad (3)$$

At the interface between liquids $x = s(t)$, the oil pressure must be equal to the pressure of the boundary water $\theta(t)$, i.e.

$$P(s(t),t) = \theta(t). \qquad (4)$$

At this boundary, in addition to condition (4), the condition of material balance must also be satisfied, from which follows the equation of motion of the interface between liquids

$$\phi_0 \frac{ds}{dt} = -\frac{k}{\mu} \frac{\partial P(s(t),\, t)}{\partial x}. \tag{5}$$

Obviously, by specifying the flow rate of the production gallery $q(t)$ and the pressure at the interface of the liquids $\theta(t)$, having solved the direct problem (1) – (5), it is possible to find the pressure distribution in the reservoir $P(x,t)$ and the position of the interface of the liquids $s(t)$ at any moment in time. An essential feature of the direct problem is the presence of a time-moving interface of the liquids, the law of movement of which is not specified a priori and must be determined in the process of solving the problem. The direct problem of the elastic-water-drive regime of reservoir development belongs to the class of single-phase Stefan problems [5,6].

However, due to the fact that the pressure at the interface of liquids $\theta(t)$ is established during the process of oil displacement, it is not available for direct measurement and it is impossible to regulate it according to a predetermined program. Therefore, in addition to the functions $P(x,t)$ and $s(t)$, the function $\theta(t)$ is also unknown. It is obvious that for the correct formulation of the initial-boundary value problem for equation (1), it is necessary to specify an additional condition instead of the missing information at the interface of the liquids $x = s(t)$. Let us assume that the change in pressure over time at the operating gallery is described by the function $f(t)$. Then, as an additional condition, we can take

$$P(0,t) = f(t). \tag{6}$$

Thus, the problem consists of determining the functions $P(x,t)$, $s(t)$ and $\theta(t)$, satisfying equations (1), (5) and conditions (2) – (4), (6). The stated problem (1) – (6) is an ill-posed problem and belongs to the class of boundary inverse problems [7–10].

It should be noted that theoretical issues related to the correctness of statements of boundary inverse problems, the existence and uniqueness of a solution to a class of boundary inverse problems for equations of mathematical physics in various functional spaces are studied in [11–13]. Many works [7, 8, 14–17] are devoted to the development and justification of computational algorithms for the numerical solution of boundary inverse problems and their numerical implementation. Currently, there is extensive literature on numerical methods for solving Stefan inverse problems [18–23]. However, most of these works are devoted to coefficient inverse problems and source problems, where the gradient iterative method is mainly used. In this paper, a computational algorithm is proposed for solving the posed inverse problem, based on discretization of the problem, preliminarily transformed by the front straightening method, and representation of the resulting system of difference equations in the form of a variational problem with local regularization.

2. Method for Solving the Problem

Assuming the existence and uniqueness of the solution to the posed inverse problem (1) – (6), we transform it using the front straightening method [6]. For this purpose, we introduce the change of variables

$$y = \frac{x}{s(t)},\ t = t, P(x,t) = P(y,t).$$

It is obvious that in this case the domain of equation (1) Ω_s is uniquely mapped onto a rectangular domain $\Omega = \{0 < y < 1,\ \ 0 < t \leq T\}$ with fixed boundaries. Then equations (1), (5) and conditions (2) – (4), (6) in the new variables take the form

$$\frac{\partial P}{\partial t} = \frac{\chi}{s^2(t)} \frac{\partial^2 P}{\partial y^2} + \frac{y}{s(t)} \frac{ds}{dt} \frac{\partial P}{\partial y}, (y,t) \in \Omega = \{0 < y < 1,\ 0 < t \leq T\}\,, \tag{7}$$

$$P(y,0)=\psi(ly), 0 \le y \le 1, \tag{8}$$

$$rh\frac{k}{\mu}\frac{1}{s(t)}\frac{\partial P(0,t)}{\partial y}=q(t), \tag{9}$$

$$P(1,t)=\theta(t), \tag{10}$$

$$P(0,t)=f(t), \tag{11}$$

$$\phi_0\frac{ds}{dt}=-\frac{k}{\mu}\frac{1}{s(t)}\frac{\partial P(1,\ t)}{\partial y}, \tag{12}$$

$$s(0)=l. \tag{13}$$

First, using the difference approximation method, we construct a discrete analogue of the differential problem (7) – (13) in a rectangular domain $\overline{\Omega}$. To do this, we introduce a uniform space-time difference grid in the domain $\overline{\Omega}$:

$$\overline{\omega}=\{(y_i,t_j):\ y_i=i\Delta y,\ \ t_j=j\Delta t,\ \ i=0,\ 1,\ 2,...,n,\ \ j=0,\ 1,\ 2,\ ...,m\}$$

with steps $\Delta y=1/n$ and $\Delta t=T/m$.
The discrete analogue of problem (7) – (13) on the grid $\overline{\omega}$can be represented as

$$\frac{P_i^j-P_i^{j-1}}{\Delta t}=\frac{\chi}{(s^2)^j}\frac{P_{i+1}^j-2P_i^j+P_{i-1}^j}{\Delta y^2}+\frac{y_i}{s^j}\frac{s^j-s^{j-1}}{\Delta t}\frac{P_{i+1}^j-P_i^j}{\Delta y}, \tag{14}$$

$$i=1,2,3,...,n-1,\ j=1,2,3,...,m,$$

$$P_i^0=\psi_i, 0\le i\le n, \tag{15}$$

$$\frac{P_1^j-P_0^j}{\Delta y}=q^js^j\frac{\mu}{rhk}\ , \tag{16}$$

$$P_n^j=\theta^j, \tag{17}$$

$$P_0^j=f^j, \tag{18}$$

$$\phi_0\frac{s^j-s^{j-1}}{\Delta t}=-\frac{k}{\mu}\frac{1}{s^{j-1}}\frac{P_n^{j-1}-P_{n-1}^{j-1}}{\Delta y}, \tag{19}$$

$$s^0=l, \tag{20}$$

where $P_i^j\approx P(y_i,t_j)$, $s^j\approx s(t_j)$, $f^j=f(t_j)$, $q^j=q(t_j)$, $\psi_i=\psi(ly_i)$, $\theta^j=\theta(t_j)$.
As can be seen, when constructing a discrete analogue of equation (7), an implicit approximation in time is used, and for equation (12), an explicit approximation in time. This approach allows the process of solving the system of difference equations (14) – (20) at each time layer j, $j=1,2,3,...,m$ to be presented as the following sequence of computational procedures.
I. Based on the given values of reservoir pressure P_n^{j-1} and P_{n-1}^{j-1}, the solution of the difference equation (19) is determined (for $j=1$, the initial conditions (15) and (20) are used), i.e. the position of the liquid interface

$$s^j=s^{j-1}-\frac{k}{\phi_0\mu}\frac{\Delta t}{s^{j-1}}\frac{P_n^{j-1}-P_{n-1}^{j-1}}{\Delta y}. \tag{21}$$

II. Taking into account the newly found value s^j, the solution to the system of difference equations (14) – (18) is determined, i.e. P_i^j and θ^j.
To solve the system of difference equations (14) – (18) for each fixed value of j, $j = 1,2,3,...,m$ we formulate it as a variational problem with local regularization [8]. To do this, we represent this system of equations in the form

$$a_i P_{i-1}^j - c_i P_i^j + b_i P_{i+1}^j = -P_i^{j-1}, i = 1,2,...,n-1, \tag{22}$$

$$P_0^j = P_1^j - q^j s^j \frac{\mu \Delta y}{rhk}, \tag{23}$$

$$P_n^j = \theta^j, \tag{24}$$

where $a_i = \dfrac{\chi \Delta t}{(s^2)^j \Delta y^2}$, $b_i = \dfrac{\chi \Delta t}{(s^2)^j \Delta y^2} + \dfrac{y_i}{\Delta y}\dfrac{s^j - s^{j-1}}{s^j}$, $c_i = a_i + b_i + 1$.
And instead of condition (18), we introduce a smoothing functional in the form

$$J(\theta^j) = \left[P_0^j - f^j\right]^2 + \alpha \left(\theta^j\right)^2 \to \min, \tag{25}$$

where α is regularization parameter.
Thus, the problem of determining θ^j at each time layer $j = 1,2,...,m$ is reduced to minimizing the smoothing functional (25) under conditions (22) – (24).

Let us assume that the solution of the system of difference equations (22) – (24) at each time layer $j = 1,2,...,m$ can be represented in the form [8, 15, 22]

$$P_i^j = V_i^j + \theta^j W_i^j,\ i = 0,1,2,...,n, \tag{26}$$

where V_i^j, W_i^j are unknown variables. Substituting the expression P_i^j into equation (22), we get

$$a_i[V_{i-1}^j + \theta^j W_{i-1}^j] - c_i[V_i^j + \theta^j W_i^j] + b_i[V_{i+1}^j + \theta^j W_{i+1}^j] = -P_i^{j-1}$$

or

$$[a_i V_{i-1}^j - c_i V_i^j + b_i V_{i+1}^j + P_i^{j-1}] + \theta^j[a_i W_{i-1}^j - c_i W_i^j + b_i W_{i+1}^j] = 0. \tag{27}$$

And substituting expressions P_i^j in (23), (24) gives

$$[V_0^j - V_1^j + q^j s^j \frac{\mu \Delta y}{rhk}] + \theta^j[W_0^j - W_1^j] = 0, \tag{28}$$

$$V_n^j + \theta^j W_n^j = \theta^j. \tag{29}$$

Obviously, the relations (27) – (29) will be automatically fulfilled under the following conditions:
a) variables V_i^j, $i = 0,\ 1,\ 2,\ ..,n$ satisfy the system of difference equation (30) – (32)

$$a_i V_{i-1}^j - c_i V_i^j + b_i V_{i+1}^j = -P_i^{j-1}, \tag{30}$$

$$V_0^j = V_1^j - q^j s^j \frac{\mu \Delta y}{rhk}, \tag{31}$$

$$V_n^j = 0; \tag{32}$$

b) variables W_i^j, $i = 0,\ 1,\ 2,\ ..,n$ satisfy the system of difference equations (33) – (35)

$$a_i W_{i-1}^j - c_i W_i^j + b_i W_{i+1}^j = 0, \tag{33}$$

$$W_0^j = W_1^j, \tag{34}$$

$$W_n^j = 1. \tag{35}$$

The obtained independent systems of difference equations (30) – (32) and (33) – (35) represent a system of linear algebraic equations with a tridiagonal matrix, the solutions of which are determined by the well-known Thomas method [8].

Having determined the values of the variables V_i^j, W_i^j, $i = n$, and substituting representation (26) into (25), we have

$$J(\theta^j) = \left[V_0^j + \theta^j W_0^j - f^j\right]^2 + \alpha \left(\theta^j\right)^2 \to \min .$$

The minimum of this functional is achieved under

$$\left[V_0^j + \theta^j W_0^j - f^j\right] W_0^j + \alpha\theta^j = 0.$$

From this we obtain a formula for determining the approximate value of the desired function $\theta(t)$ under

$$t = t_j,$$

$$\theta^j = \frac{W_0^j (f^j - V_0^j)}{(W_0^j)^2 + \alpha}. \tag{36}$$

After determining the value θ^j, one can use formula (26) to calculate the approximate values of the desired function $P(y,t)$ on the time layer j, i.e. P_i^j, $i = 0, 1, 2, ..., n-1$.
Thus, the computational algorithm for the numerical solution of the inverse problem (8) – (13) for restoring the pressure distribution and the position of the liquid interface in the reservoir at each time layer j, $j = 1, 2, ..., m$, consists of the following stages:
1) The position of the interface of liquids is determined by formula (21);
2) in parallel, solutions of two independent linear systems of difference equations (30) – (32) and (33) – (35) are determined with respect to auxiliary variables V_i^j, W_i^j, $i = 0, 1, 2, ..., n$;
3) according to formula (36) the pressure at the interface between liquids is determined, i.e. θ^j;
4) according to formula (26) the pressure values at the remaining nodal points are calculated, i.e. P_i^j, $i = 0, 1, 2, ..., n-1$.

Thus, the proposed numerical method allows one to determine the position of the liquid interface and the pressure distribution in the reservoir in each time layer.

3. Results of Numerical Experiments

To test the efficiency of the practical application of the proposed computational algorithm, numerical experiments were carried out for a model rectangular oil reservoir with the following characteristics: length of the formation $l = 100$ m; thickness of the formation h=10 m; width of the formation $r = 100$ m; absolute permeability coefficient of the formation $k = 0,5 \cdot 10^{-12}$ m^2; initial formation pressure 180 atm; production gallery flow rate $q(t) = 240$ m^3/day; dynamic viscosity of oil $\mu = 3 \cdot 10^{-3}$ Pa·s; coefficient of piezoconductivity of the formation $\chi = 5$ m^2/s; porosity coefficient of the formation $\phi_0 = 0,3$.

A numerical experiment covering a period of $T = 40$ days of development of the specified oil reservoir was carried out according to the following scheme:
1. The pressure at the interface between the liquids is specified and the solution to the direct problem (7) – (10), (12), (13) is determined.
2. The found dependence $f(t) = P(0,t)$ was taken as exact data for the numerical solution of the inverse problem of recovery $\theta(t)$.
In the numerical experiments, unperturbed and perturbed input data were used. When using unperturbed input data, the value of the regularization parameter α was taken to be zero. To perturb the input data, the following relationship was used:

$$\tilde{f}(t) = f(t) + \delta\sigma(t)f(t)\,,$$

where the term $\delta\sigma(t)f(t)$ models different error levels for the input data $f(t)$; $\sigma(t)$ is a random variable modeled using a random number generator; δ is the error level. In this case, the value of the regularization parameter was determined in accordance with the residual principle [7, 8].

The calculations were carried out on a uniform space-time difference grid with steps of $\Delta x = 5$ m, $\Delta t = 60s$. The results of numerical experiments on the reconstruction of two functions $\theta_1(t) = 180$atm and $\theta_2(t) = 270e^{-t/T}$atm, using unperturbed and perturbed input data, are presented in Table 1.

Table 1

Results of numerical experiments on pressure recovery at the interface of liquids

t day	$\theta_1(t) = 180\,atm$		$\theta_1(t) = 270e^{-t/T}atm$	
	θ_1^t, $\bar{\theta}_1$	$\tilde{\theta}_1$	θ_2^t, $\bar{\theta}_2$	$\tilde{\theta}_2$
2	180	183,14	257,35	262,03
4	180	181,85	245,29	247,93
6	180	180,29	233,79	234,22
8	180	177,55	222,83	219,78
10	180	178,56	212,39	210,69
12	180	179,81	202,44	202,24
14	180	176,35	192,95	189,02
16	180	177,91	183,91	181,77
18	180	179,27	175,29	174,58
20	180	180,19	167,07	167,24
22	180	184,00	159,24	162,73
24	180	184,58	151,78	155,57
26	180	184,58	144,67	148,26
28	180	179,03	137,89	137,14
30	180	181,00	131,42	132,11
32	180	180,75	125,26	125,75
34	180	179,38	119,39	118,97
36	180	176,99	113,80	111,94
38	180	178,68	108,46	107,64
40	180	178,69	103,38	102,64

In it t is time; θ_1^t, θ_2^t are the exact values of the functions $\vartheta\theta_1(t)$ and $\theta_2(t)$; $\bar{\theta}_1$, $\bar{\theta}_2$ are the calculated values of the functions $\vartheta\theta_1(t)$ and $\theta_2(t)$ with unperturbed input data; $\tilde{\theta}_1$, $\tilde{\theta}_2$ are the calculated value of the functions $\vartheta\theta_1(t)$ and $\theta_2(t)$ with perturbed input data.
As the results of numerical experiments show, when using unperturbed input data, the desired function $\theta(t)$ is restored absolutely accurately. Therefore, the exact and calculated values of the pressure at the interface of liquids at each moment of time are presented in the same column of the table (the second and fourth columns of Table 1).
Consequently, the pressure distribution in the model layer under consideration is also restored with absolute precision. Table 2 shows the pressure distribution at time t=40 day and the displacement of the liquid interface during this period.

When using perturbed input data, in which the error has a fluctuating nature, a weak dependence of the restoration of the desired function $\theta(t)$ on this error appears (the third and fifth columns of Table 1). To perturb the input data, the error level was used $\delta = 0,02$. However, in this case, for a given error level, the maximum relative error in restoring the values of the desired function $\theta(t)$ does not exceed 2,7%. The value of the regularization parameter was $0,0032$.

Analysis of the results of numerical experiments shows that the proposed computational algorithm ensures the stability of the solution to errors in the input data.

Table 2

Dynamics of fluid interface movement and pressure distribution in the reservoir at $t = 40$ day

t, day	$s_1(t)$, m	$s_2(t)$, m		x, m	$P_1(x,t)$,atm	$P_2(x,t)$,atm
0	100	100		0	168,66	92,05
2	98,42	98,33		5	169,23	92,62
4	96,81	96,75		10	169,40	93,18
6	95,21	95,16		15	170,36	93,75
8	93,61	93,57		20	170,93	94,32
10	92,01	91,98		25	171,50	94,88
12	90,41	90,39		30	172,06	95,45
14	88,81	88,80		35	172,63	96,02
16	87,21	87,20		40	173,20	96,58
18	85,61	85,61		45	173,77	97,15
20	84,01	84,02		50	174,33	97,72
22	82,41	82,43		55	174,90	98,28
24	80,81	80,83		60	175,47	98,85
26	79,21	79,24		65	176,03	99,42
28	77,61	77,64		70	176,60	99,98
30	76,01	76,05		75	177,17	100,55
32	74,41	74,45		80	177,73	101,12
34	72,81	72,86		85	178,30	101,18
36	71,21	71,26		90	178,87	102,25
38	69,61	69,66		95	179,43	102,82
40	68,01	68,07		100	180	103,38

Conclusion

Within the framework of a one-dimensional model of elastic-water-driven development mode, the problem of identifying the main hydrodynamic parame-ters of a reservoir based only on information obtained from a gallery of production wells is considered. For the numerical solution of the problem, a computational algorithm based on the use of front straightening methods, difference approxima-tion and local regularization is proposed. Testing of the proposed computational algorithm on data from a model oil reservoir confirms the high accuracy of determining the dynamics of pressure distribution and the movement of the liquid interface during the development of an oil reservoir.

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ЧИСЛЕННАЯ ИДЕНТИФИКАЦИЯ ГИДРОДИНАМИЧЕСКИХ ПАРАМЕТРОВ ПЛАСТА ПРИ УПРУГОВОДОНАПОРНОМ РЕЖИМЕ РАЗРАБОТКИ

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Рассматривается процесс разработки нефтяного пласта в упруговодонапорном режиме. Предполагается, что вытеснение нефти краевой водой происходит полностью и в пласте образуется четкая граница раздела двух жидкостей, которая движется по заранее неизвестному закону. В рамках одномерной модели упруговодонапорного режима разработки поставлена задача идентификации основных гидродинамических параметров пласта, т.е. давления на границе раздела жидкостей, распределения давления в пласте и положения границы раздела жидкостей, только на основании информации, полученной из галереи эксплуатационных скважин. Поставленная задача относится к классу граничных обратных задач.

Применяя методы выпрямления фронтов и разностной аппроксимации, поставленная задача сводится к решению системы разностных уравнений. Для решения системы разностных уравнений предлагается специальное представление, предварительно записавая ее в виде вариационной задачи с локальной регуляризацией. В результате получены явная формула для определения приближенного значения давления на границе раздела жидкостей и рекуррентные формулы для определения распределения давления и положения границы раздела жидкостей в пласте на каждом временном слое. На основе предложенного вычислительного алгоритма были проведены численные эксперименты для модельного нефтяного пласта.

Ключевые слова: упруговодонапорный режим разработки; граничная обратная задача; метод выпрямления фронтов; локальная регуляризация; разностный метод.

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