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AN EMPIRICAL STUDY OF THE QUALITY OF THE CYCLE MERGING ALGORITHM FOR THE MAXIMUM TRAVELING SALESMAN PROBLEM

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The maximum traveling salesman problem (MAX TSP) is a mathematical optimization problem that requires the construction of a Hamiltonian cycle with the highest possible sum of edge weights. It finds applications in bioinformatics, coding, logistics, and numerous other fields. Despite the existence of theoretical bounds on the accuracy of approximation algorithms, their practical behavior remains insufficiently explored. In this study, we conduct an empirical analysis of the Cycle Merging Algorithm (CMA) for solving MAX TSP. The CMA is a greedy heuristic based on the sequential merging of cycles in a maximum-weight 2-factor. Our computational experiments, carried out on instances ranging from 100 to 3000 vertices, evaluate the accuracy of the CMA solutions relative to an upper bound determined by solving an assignment problem, as well as the algorithm's computational efficiency. A significant contribution of this work is the construction of a regression model that describes the dependency of the relative error estimate on the number of vertices for metric instances. The model demonstrates that the relative error decreases according to a power-law relationship, and the analysis confirms that the CMA consistently outperforms its guaranteed theoretical bound. The results indicate that the Cycle Merging Algorithm is a powerful heuristic for MAX TSP, providing high-quality solutions and computational efficiency in practice. Future research directions include optimizing the cycle merging strategy, developing hybrid algorithms, and implementing GPU-based version to enhance scalability.

Keywords: maximum traveling salesman problem; approximation algorithms; computational experiment; regression analysis; algorithmic complexity.

Introduction

The Traveling Salesman Problem (TSP) is one of the fundamental problems of combinatorial optimization studied since the middle of the 20th century. It is formulated as follows: given a set of cities and distances between them, it is necessary to find the shortest Hamiltonian cycle that visits each city exactly once. The classical traveling salesman problem belongs to the NP-hard [1] problem, which makes its exact solution in polynomial time impossible, unless the hypothesis $P = NP$ is fulfilled. Therefore, approximate algorithms that guarantee to find solutions with error bounded from above are widely used to solve problems of practical scales.

A variation of the maximal traveling salesman problem (MAX TSP) involves finding the Hamiltonian cycle with the largest sum of edge weights. Despite the apparent symmetry between minimization and maximization problems, MAX TSP has a number of specific properties that distinguish it from the classical TSP. In particular, in practice it is used for such tasks as reconstruction of evolutionary trees in bioinformatics [2], information coding, data packing [3], routing with profit maximization in mind [4] and other.

An important feature of the maximal traveling salesman problem is the availability of polynomial approximate algorithms with high guaranteed accuracy estimates. Thus, for arbitrary asymmetric graphs an approximation of 2/3 [5] is proved, and for the metric case a higher bound of 7/8 [8] is obtained. Table summarizes the approximation factors of currently best polynomial-time algorithms for the traveling salesman problem with different types of cost matrix.

Table 1

Accuracy of algorithms for MAX TSP

Type of cost matrix	Approximation factor
arbitrary asymmetric case [5]	2/3
arbitrary symmetric case [6]	7/9
asymmetric metric case [7]	35/44
metric case [8]	7/8

However, these bounds are not always achieved in practice, and the quality of the approximate solutions may significantly outperform the theoretical estimates. This raises the need to study the behavior of specific algorithms on real data.

One of the simplest and most natural heuristic algorithms for MAX TSP is the Cycle Merging Algorithm (CMA), which is based on sequentially merging cycles in a 2-factor of maximum weight. This algorithm bears a conceptual resemblance to the greedy Karp–Steele patching heuristic [9] for which, in the case of solving a metric maximum problem, V. Shenmaier proved asymptotic accuracy [10]. However, detailed numerical studies of the CMA evaluating its practical accuracy and behavior on different types of data have been lacking so far.

Our study is organized as follows. In the first section, we describe the CMA and its theoretical properties, including its worst-case guarantee of at least 5/6 of the optimal weight. In the second section, we detail the experimental design, including the generation of cost matrices (metric, symmetric, and asymmetric) and the evaluation metrics used. The third section presents our computational experiments, which span problem sizes from 100 to 3000 vertices, and includes both accuracy analysis: comparing the CMA solutions with an upper bound obtained from the Hungarian algorithm and computational efficiency profiling. In section five, we construct a regression model showing the asymptotic behavior of the relative error of the CMA as the number of vertices increases. Finally, we present a comparison of the model values for a large number of vertices with theoretical bounds.

1. Cycle Merging Algorithm

In this section, we describe the Cycle Merging Algorithm (CMA) for solving the maximum traveling salesman problem (TSP) [11–13]. A computational and statistical analysis of its accuracy will be discussed in subsequent sections. The algorithm follows the general idea of the modified Karp–Steele patching heuristic [9].

The CMA algorithm operates by sequentially merging cycles within an initial cycle cover obtained as a maximum-weight 2-factor of a given complete graph $G = (V, E)$, where V represents the set of vertices and E denotes the set of edges.

1.1. Algorithm Overview

1. Initial 2-Factor Construction: a maximum-weight 2-regular subgraph is found, forming a cycle cover. This corresponds to solving an assignment problem (AP) on a complete bipartite graph, which can be computed in $O(n^3)$ time.

2. Cycle Merging Phase: if the obtained 2-factor consists of multiple cycles, the algorithm iteratively selects and merges two cycles that maximize the increase in total weight.

3. Termination: the process continues until a single Hamiltonian cycle remains.

1.2. Theoretical Approximation Guarantees

The following theorems establish formal guarantees for the CMA:

Theorem 1. *The computational complexity of the CMA does not exceed $O(|V|^3)$.*

Theorem 2. *Let W_{opt} be the optimal value of the metric maximum TSP, W_C be the weight of the maximum 2-factor, and W_{alg} be the weight of the solution produced by the CMA. Then:*

$$\frac{W_{alg}}{W_{opt}} \geq \frac{5}{6}. \tag{1}$$

The bound $W_{alg}/W_C = 5/6$ is achievable.

The proofs of both theorems are given in [14]. This bound guarantees that the algorithm performs within at least 83,3% of optimal solutions for any metric instance of TSP. A more refined asymptotic accuracy bound was established by V. Shenmaier [10] for a similar greedy patching heuristic:

$$\frac{W_{alg}}{W_{opt}} \geq 1 - \frac{7}{3}n^{-1/5}. \tag{2}$$

This result suggests that for sufficiently large problem sizes, the accuracy of such patching heuristics converges to 100%. In Section 5 we compare our empirical accuracy results with these theoretical guarantees.

2. Experiment Description

To evaluate the performance and accuracy of the proposed Cycle Merging Algorithm (CMA), a C++ software implementation was developed and tested on various types of cost matrices. The primary goal of the experiments was to verify the theoretical results obtained for the metric maximum traveling salesman problem and to assess the practical efficiency of the CMA under different cost matrix structures.

2.1. Cost Matrix Generation

The algorithm was tested on three different types of cost matrices:

- *Euclidean metric:* the cost matrix was generated using random points in a two-dimensional plane, where edge weights were computed as the Euclidean distances between points.
- *Symmetric random:* each entry c_{ij} in the cost matrix was assigned a random integer value in the range $[1, 10^5]$, ensuring symmetry ($c_{ij} = c_{ji}$).
- *Asymmetric random:* entries were randomly generated in the same range, but without enforcing symmetry.

The number of vertices n varied from 100 to 3000 in increments of 100, providing a broad spectrum of test cases.

2.2. Evaluation Metrics

To assess the quality and efficiency of the CMA, the following metrics were considered:

- Relative error estimate: defined as the ratio of the difference between the upper bound estimate and the obtained solution to the obtained solution itself:

$$\text{Relative Error} = \frac{W_{\text{upper}} - W_{\text{alg}}}{W_{\text{alg}}}, \tag{3}$$

  where W_{upper} is the upper bound given by the solution to the assignment problem, and W_{alg} is the weight of the cycle produced by the CMA.
- Execution time: the average computational time required to obtain a solution for each problem instance was measured.

All reported values represent averages over 100 independent trials to ensure statistical reliability.

2.3. Experimental Setup

The tests were conducted on a machine with the following hardware and software specifications:

- Processor: AMD A10-4600M APU with Radeon™ HD Graphics, 2.30 GHz.
- Memory: 12 GB DDR3 RAM.
- Storage: 128 GB SSD.
- Operating System: Windows 10 Pro 64-bit.
- Compiler: Microsoft Visual Studio 2022 (C++20 standard).

3. Results of the Computational Experiment for the Maximal Traveling Salesman Problem

3.1. Relative Error Estimate Analysis for Different Cost Matrices

The relative error estimate of the computed solutions varies across different cost matrix types, as shown in Fig. 1. The asymmetrical cost matrix exhibits the lowest relative error estimate across all problem sizes, with values decreasing from approximately 0,0107 for 100 vertices to 0,00034 for 3000 vertices. This suggests that asymmetrical instances allow for better optimization and more stable solutions.

The symmetrical cost matrix shows consistently higher relative error estimates, starting at 0,0374 for 100 vertices and decreasing to 0,00535 for 3000 vertices. This behavior indicates that symmetrical matrices introduce additional constraints that affect the accuracy of the computed solutions.

The Euclidean cost matrix demonstrates intermediate relative error estimate values, ranging from 0,0198 (100 vertices) to 0,00054 (3000 vertices). The error decreases rapidly for smaller problem sizes and stabilizes for larger instances, suggesting that Euclidean instances benefit from natural geometric properties that improve convergence.

Overall, the relative error estimate follows a decreasing trend across all cases, indicating that larger problem instances result in more accurate solutions relative to their optimal counterparts. However, the rate of error reduction is steeper for asymmetrical and Euclidean matrices compared to symmetrical ones.

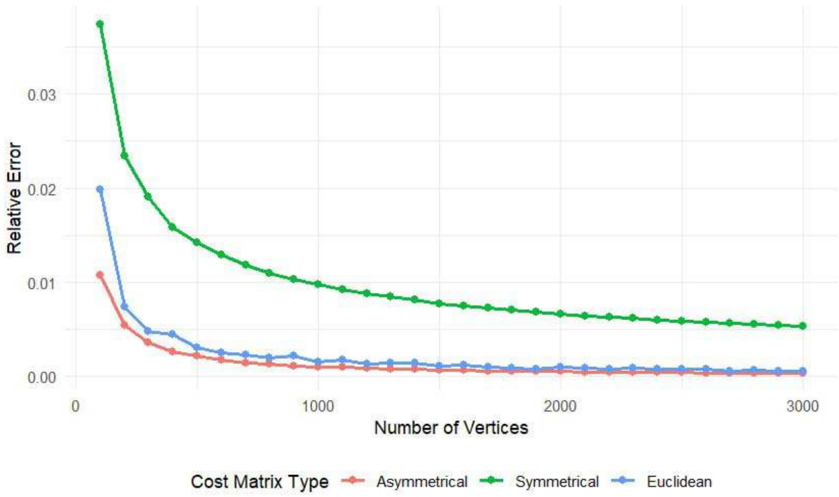

Fig. 1. Relative error estimate for different cost matrices

3.2. Execution Time Analysis for Different Cost Matrices

The execution time for solving TSP instances varies significantly depending on the cost matrix type, as illustrated in Fig. 2. The Euclidean cost matrix demonstrates the fastest execution times, ranging from 0,0018 s for 100 vertices to 5.17 s for 3000 vertices. This suggests that the natural structure of Euclidean instances allows for more efficient computations.

The asymmetrical cost matrix exhibits moderate execution times, increasing from 0,0078 s (100 vertices) to 9,23 s (3000 vertices). The execution time grows approximately linearly, indicating efficient processing despite the increased complexity compared to Euclidean matrices.

The symmetrical cost matrix leads to the highest execution times, growing from 0,0096 s for 100 vertices to 62,24 s for 3000 vertices. The sharp increase in computational time suggests that solving symmetrical instances introduces additional computational overhead, likely due to the increased number of equivalent solutions and the added complexity in cycle merging.

These results highlight that the specificity of cost matrix significantly affects computational efficiency. While Euclidean instances are processed the fastest, symmetrical matrices impose additional constraints that lead to longer execution times.

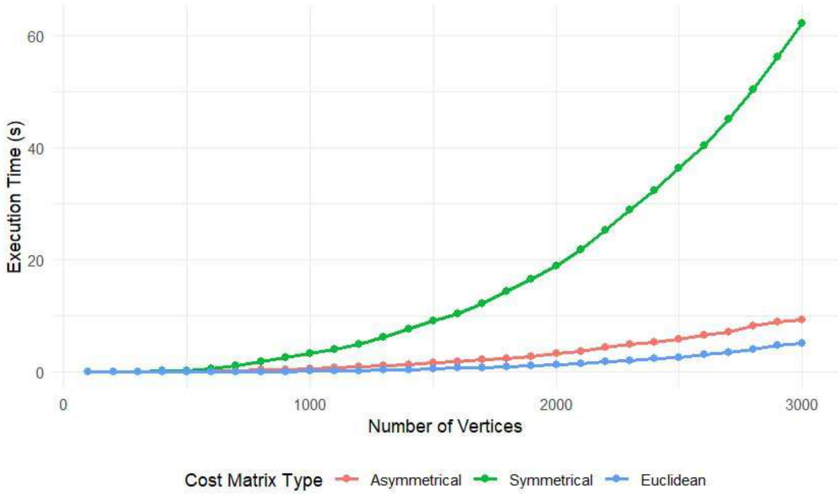

Fig. 2. Execution time for different cost matrices

3.3. Comparison with Theoretical Complexity and Approximation Guarantees

The computational complexity of the Cycle Merging Algorithm (CMA) is theoretically bounded by $O(|V|^3)$, as established in Theorem 1. This prediction aligns well with the observed execution times for different cost matrices. Specifically, for Euclidean instances, the experimental results show near-cubic growth. This confirms that the CMA implementation behaves as expected in practice, maintaining computational feasibility even for large instances.

Furthermore, Theorem 2 guarantees a worst-case approximation ratio of at least 5/6 for the metric maximum traveling salesman problem. For Euclidean instances, the relative error results indicate that the computed solutions are well within this bound. The observed relative error estimates decrease as problem size increases, reaching as low as 0,00054 for 3000 vertices. This suggests that in practical cases, the CMA often performs better than the theoretical bound, achieving near-optimal solutions.

Additionally, the comparison of Euclidean execution times with asymmetrical and symmetrical cases highlights the efficiency of solving metric TSP instances. The lower execution time for Euclidean instances suggests that geometric properties of the problem structure may contribute to improved computational performance, reducing unnecessary edge evaluations and simplifying cycle merging.

Overall, the experimental findings confirm that the CMA adheres to its theoretical complexity bound and approximation guarantee while demonstrating strong practical performance, particularly for Euclidean instances.

4. Statistical Analysis of the CMA Accuracy

The statistical study aims to analyze the relationship between the number of vertices in the metric maximum traveling salesman problem (TSP) and the accuracy of solutions produced by the Cycle Merging Algorithm (CMA). The primary objective is to determine the functional form of this relationship and assess the validity of the obtained regression model.

4.1. Exploratory Data Analysis

Exploratory analysis was performed to visualize the empirical data and make initial assumptions about the regression model. The pattern observed in the Fig. 1 for Euclidean instances of the TSP implies a power-law relationship between the number of vertices n and the mean relative error estimate y, described by:

$$y = a \cdot n^b. \tag{4}$$

To apply linear regression, both variables were transformed using logarithms:

$$\log y = \log a + b \log n. \tag{5}$$

4.2. Regression Model Construction and Assumptions Validation

A linear regression model was fitted to the logarithmically transformed data using the least squares method (LSM). To ensure the validity of the model, standard assumptions – independence, homoscedasticity, and normality of residuals – were tested.

The Durbin–Watson test, the Breusch–Pagan test, and the global validation of linear model assumptions (GVLMA) did not reveal any significant violations (Table 2). The obtained statistics and p-values confirm that the model satisfies the Gauss–Markov assumptions. Therefore, the parameter estimates can be considered reliable, and the model is suitable for further analysis.

Table 2

Global validation of linear model assumptions

Test Component	Statistic	p-value
Global Stat	5,8085	0,21391 (Acceptable)
Skewness	0,3769	0,53926 (Acceptable)
Kurtosis	1,7497	0,18591 (Acceptable)
Link Function	2,7340	0,09823 (Acceptable)
Heteroscedasticity	0,9478	0,33028 (Acceptable)

4.3. Model Evaluation and Interpretation

After back-transformation, the regression equation in the original variables takes the form:

$$y = 1,290 \cdot n^{-0,95752}. \tag{6}$$

Equation (6) describes a decreasing relationship between the relative error estimate and the number of vertices. The negative exponent $-0,95752$ indicates that the error decreases as the problem size increases. The coefficient $1,290$ defines the scale of the relationship and corresponds to the model's predicted error value when $n = 1$ (in the theoretical interpretation of the model).

The coefficient of determination ($R^2 = 0,9819$) confirms a strong correlation between n and y, supporting the validity of the model.

Key statistical metrics:

- R^2 (explained variance): 0,9819 (high model fit).
- F-statistic: 1520 ($p < 2,2e - 16$), indicating strong model significance.
- Confidence intervals: both coefficients are highly significant ($p < 0,001$), as confirmed by *Student's t-test*.

A graphical comparison of empirical and model data is shown in Fig. 3.

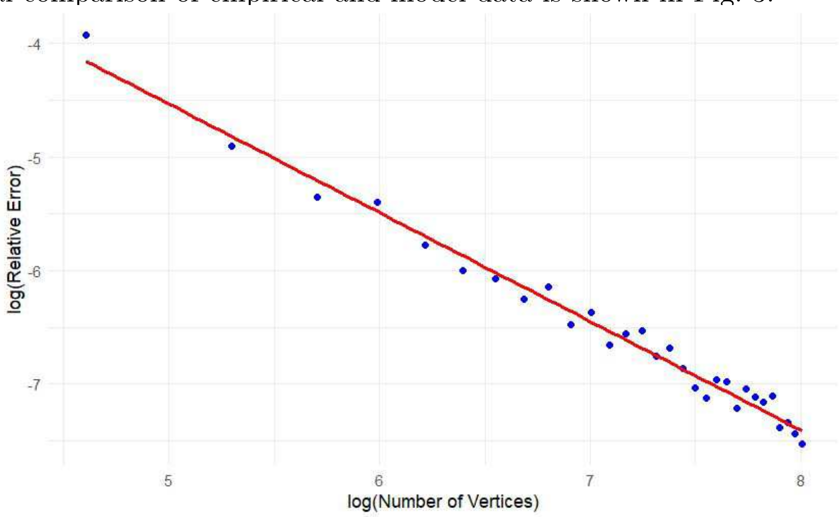

Fig. 3. Fitted power-law model on the log-transformed data

The residual diagnostic plots in Fig. 4 confirm that the model satisfies the assumptions of normality and homoscedasticity.

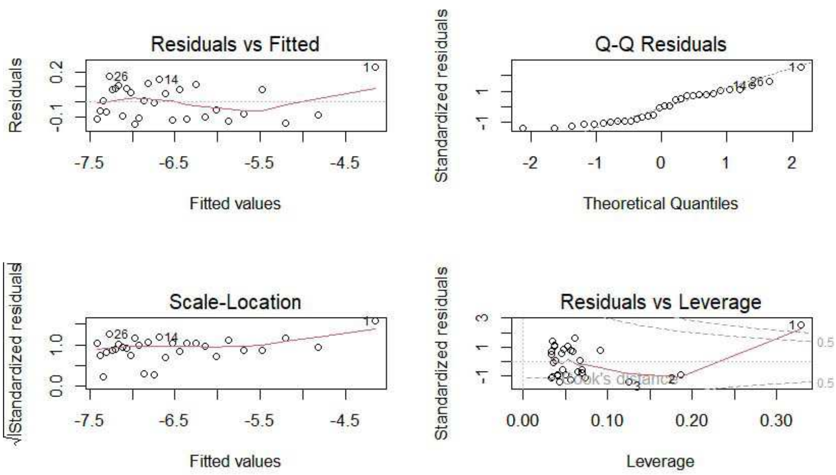

Fig. 4. Residual diagnostics of the regression model

4.4. Prediction for Large Instances

To extrapolate the results, the model was used to predict accuracy for problem sizes up to 10,000 vertices (Fig. 5). The trend suggests a continued decrease in relative error.

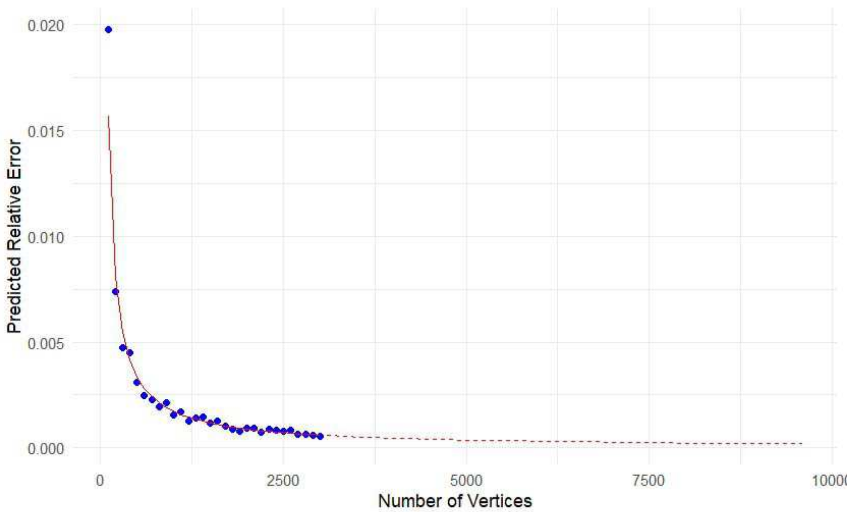

Fig. 5. Prediction of relative error estimate for large instances

The statistical analysis confirms the asymptotic efficiency of the CMA and its practical applicability to large-scale metric TSP instances.

5. Comparison with Theoretical Bounds

To analyze the long-term behavior of the Cycle Merging Algorithm (CMA), we compare its empirical error with two theoretical bounds:

- CMA theoretical bound: the worst-case relative error of the CMA does not exceed $1 - 5/6 = 1/6$ (Theorem 2).

- Shenmaier bound: the greedy patching heuristic analyzed in [10] provides the following upper bound on error:

$$\text{Error} \leq \frac{7}{3} \cdot n^{-1/5}. \tag{7}$$

Fig. 6 illustrates the empirical error, the fitted regression model, and the theoretical bounds up to $n = 10^6$.

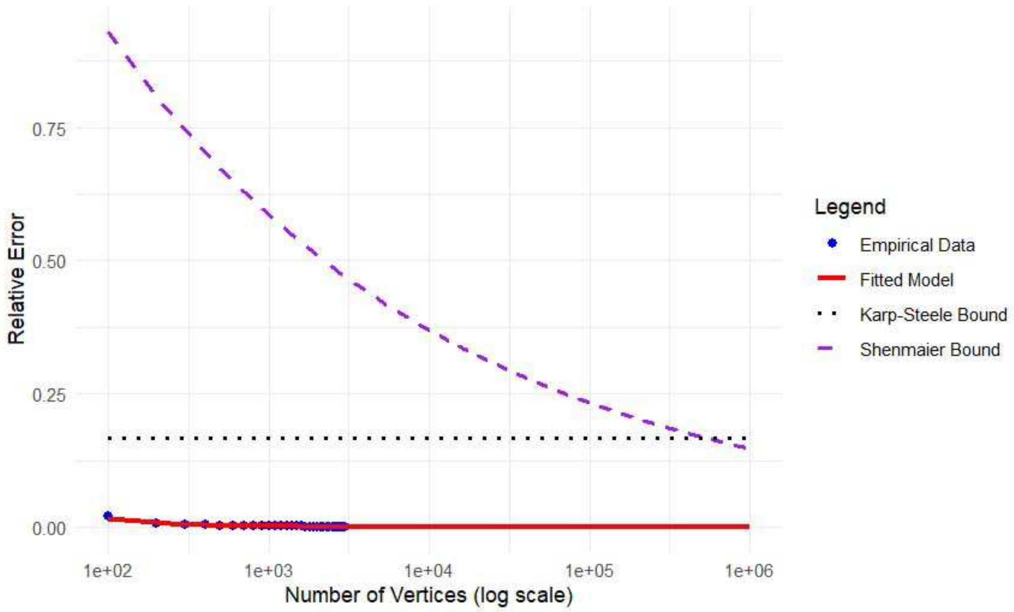

Fig. 6. Comparison of empirical CMA error with theoretical bounds

Observations and Discussion

- The empirical error follows a power-law trend, modeled as (6).
- The CMA bound (1/6) is a conservative upper bound.
- The Shenmaier bound predicts a gradual decay.
- The empirical error of the CMA remains below both theoretical bounds for practical problem sizes, demonstrating better-than-guaranteed performance up to 10^6 vertices.

These findings confirm that the CMA algorithm maintains high solution accuracy for metric maximum TSP instances, achieving significantly better performance than worst-case theoretical predictions in practical scenarios.

Conclusion

This study presents an empirical evaluation of the Cycle Merging Algorithm (CMA) for solving the maximum traveling salesman problem. The analysis focused on both solution accuracy and computational efficiency across different cost matrix structures. The results demonstrate that the CMA consistently provides high-quality solutions, with relative error estimates decreasing as the problem size grows. Among the three considered cost structures, the algorithm performed fastest on metric instances, while symmetric instances required the longest computation time. The parallel implementation of the CMA further improves its efficiency, making it suitable for large-scale problems.

A key aspect of this study was comparing the empirical accuracy of the CMA with theoretical performance guarantees. The CMA bound provides a lower guarantee of at least 5/6 of the optimal solution weight, while Shenmaier's theoretical bound suggests that greedy patching heuristics tend toward optimality as the problem size increases. Regression analysis revealed that the empirical accuracy follows a power-law trend, closely approximated by a function of the form $W_{alg}/W_{opt} = \min(1, 0,2547 \cdot n^{0,9575})$. The CMA demonstrates superior accuracy compared to theoretical guarantees for practical problem sizes, maintaining high performance even for large instances. Only in extremely large cases ($n \gg 10^6$) does Shenmaier's bound surpass the empirical accuracy of the CMA.

These findings confirm that the Cycle Merging Algorithm is a computationally efficient and practical approach to solving the maximum TSP. However, further improvements can be made to reduce the approximation error and enhance performance. Future research may explore hybrid approaches that combine the CMA with metaheuristic techniques, and distributed or GPU-based implementations to scale the algorithm for massive instances. The empirical results obtained in this study suggest that the CMA remains a competitive approximate algorithm for combinatorial optimization problems in practical applications.

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ЭМПИРИЧЕСКОЕ ИССЛЕДОВАНИЕ КАЧЕСТВА АЛГОРИТМА СОЕДИНЕНИЯ ЦИКЛОВ ДЛЯ ЗАДАЧИ КОММИВОЯЖЕРА НА МАКСИМУМ

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Задача коммивояжера на максимум – это задача комбинаторной оптимизации, заключающаяся в построении гамильтонова цикла с наибольшей суммой весов ребер. Она применяется в биоинформатике, кодировании, логистике и других областях. Несмотря на наличие теоретических границ точности приближенных алгоритмов, их реальное поведение остается недостаточно изученным. В данной работе приводится эмпирический анализ алгоритма соединения циклов (Cycle Merging Algorithm, CMA) для решения задачи коммивояжера на максимум. CMA является жадной эвристикой, основанной на последовательном объединении циклов в 2-факторе максимального веса. В ходе вычислительного эксперимента (на наборах от 100 до 3000 вершин) исследуется точность решений CMA относительно верхней границы, определяемой как решение задачи о назначении, а также вычислительная эффективность метода. Особый вклад работы заключается в построении регрессионной модели, описывающей зависимость оценки относительной погрешности от числа вершин для метрических экземпляров задачи. Модель показывает, что относительная погрешность убывает по степенному закону, а анализ подтверждает, что CMA стабильно превосходит гарантированную теоретическую границу. Полученные результаты свидетельствуют о том, что алгоритм соединения циклов является мощной эвристикой для задачи коммивояжера на максимум, обеспечивая высокое качество решений и вычислительную эффективность на практике. Перспективными направлениями дальнейшей работы являются оптимизация стратегии объединения циклов, разработка гибридных алгоритмов реализация GPU-версии для улучшения масштабируемости.

Ключевые слова: задача коммивояжера на максимум; приближенные алгоритмы; вычислительный эксперимент; регрессионный анализ; сложность алгоритма.

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