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A BICRITERIA FISH WAR GAME WITH ASYMMETRIC ENVIRONMENTAL CONCERN

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We consider spatial dynamic fishery management problem taking into account the resource migration process between an open-access fishing area and no-take marine protected area. The introduced extension of a standard single-criterion fish war game implies that each player aims to maximize simultaneously two performance criteria which present an economic benefit and an environmental conservation goal respectively. The paper studies a two-player game with asymmetric environmental concern of the players. Using dynamic programming method, we derive non-cooperative and cooperative solution in feedback stationary strategies for an infinite-horizon multistage bicriteria game with two asymmetric players. The numerical simulation is provided to study the solutions properties.

Keywords: renewable resource extraction; multi-objective game; dynamic game; fisheries; Pareto equilibria; cooperative solution; tragedy of the commons.

Introduction

The dynamic models of renewable resource extraction have been extensively studied by the specialists in applied mathematics, economics and ecology (see, e.g., [1–7]). The main problem arising in the process of dynamic use of a common resource by several competing actors was revealed in the seminal paper [8]. Several obvious examples of the "tradegy of the commons" problem in the practice of marine fishery, leading to a significant decrease or even extinction of some species of commercial fish, are well known (see, for instance, [4, 8]).

In recent decades, increasing attention has been paid to studying the rational organization of the marine fishery, taking into account the need to preserve the environment for future generations and keep long-term fishery [9]. Among the tools proposed by experts to implement a sustainable cooperation of the players involved in the dynamic extraction of a common renewable resource, we note the following: equilibria based on incentive strategies, self-enforcing environmental agreements, regulatory tax-subsidy systems, imposing a temporary moratorium [4, 10], establishment and enlargement of special marine protected areas (MPAs) or the "fish banks" [2, 4, 9], payoff distribution procedures [11], considering the players' environmental concern as a special goal function or performance criterion [12, 13], etc.

A promising bicriteria extension of the standard fish war dynamic game model [14] with fish bank and migration [4, 6, 7] was recently introduced in [15]. Non-cooperative and cooperative solutions are derived and studied for bicriteria dynamic game with n symmetric players. However, the dynamic competitive models of renewable-resource extraction with asymmetric players (see, e.g., [1, 16, 17]) have to be studied as well.

In the paper, we focus on the bicriteria multistage infinite-horizon fish war model with two asymmetric players who have different values of the environmental concern. A standard performance criterion representing the economic utility is supplemented with an additional performance criterion which has an environmental sense, that is why a multicriteria approach (see, e.g., [12, 13, 16, 18–21]) is employed to explore the introduced bicriteria dynamic fishery management model.

The contributions of this paper are twofold:

- We derive non-cooperative (subgame perfect Pareto equilibrium) and cooperative solutions for a two player bicriteria fish war game model with MPA and migration when the players have different values of the environmental concern (asymmetric players).

- Using numerical simulation, we explore how the level of the players' asymmetry affects the solutions properties.

The remainder of the paper is organized as follows. In Section 1, we briefly specify the bicriteria game model with two asymmetric players. The non-cooperative solution (subgame-perfect Pareto equilibrium in stationary strategies) is derived in Section 2. The optimal harvesting strategies under a cooperative scenario are provided in Section 3. In Section 4, we present the results of numerical simulation for different levels of the players' asymmetry. We briefly conclude in Section 5.

1. The Model

A multicriteria extension of a standard fish war n-player dynamic game model with fish bank and migration [4, 6, 7] was introduced in [15]. For the convenience of the reader, we provide below a brief specification of this model, following [4, 6, 7, 15].

Namely, we assume a fish migration process between two marine zones with different fishery regimes – a no-take reserve (MPA) or a "fish bank" S of size $s \in (0,1)$ and an open access fishing zone Q of size $q = 1 - s$. The "state" variables x_t and y_t denote resource stocks at time $t = 0, 1, \ldots, \infty$ in zones Q and S respectively, $\alpha_x \in (0,1)$ and $\alpha_y \in (0,1)$ denote the natural growth parameters in these zones. Parameter $\beta_x \in (0,1)$ represents a fish migration rate from zone S to zone Q while $\beta_y \in (0,1)$ denotes a migration rate in opposite direction, i.e., from Q to S.

In the paper, we employ the following resource dynamics for the case without any human fishing activity (see [4, 6, 7]):

$$\begin{cases} x_{t+1} = x_t^{\alpha_x} \cdot \left(\frac{y_t}{x_t}\right)^{(1-s)\beta_x}, \\ y_{t+1} = y_t^{\alpha_y} \cdot \left(\frac{x_t}{y_t}\right)^{s\beta_y}, \end{cases} \tag{1}$$

where x_0 and y_0 are some given positive initial stocks.

Following [4, 15], we adopt the following assumptions about the environmental parameters:

$$\alpha_x > \alpha_y > \beta_y > \beta_x, \alpha_x + \beta_x < 1, \alpha_y + \beta_y < 1. \tag{2}$$

Now suppose that two players take fish in area Q, and $u_{it} \geq 0$, $i = 1,\ 2$, denotes the catch of the i-th player at time t,

$$u_t = \sum_{i=1}^{2} u_{it} < x_t. \tag{3}$$

Then the fish stocks dynamics takes the following form

$$\begin{cases} x_{t+1} = (x_t - u_t)^{\widehat{\alpha}_x} \cdot y_t^{\widehat{\beta}_x}, \\ y_{t+1} = y_t^{\widehat{\alpha}_y} \cdot (x_t - u_t)^{\widehat{\beta}_y}, \end{cases} \tag{4}$$

where

$$\begin{cases} \widehat{\alpha}_x = \alpha_x - (1-s)\beta_x, \quad \widehat{\beta}_x = (1-s)\beta_x, \\ \widehat{\alpha}_y = \alpha_y - s\beta_y, \quad \widehat{\beta}_y = s\beta_y. \end{cases} \tag{5}$$

It follows from (2), that $0 < \widehat{\alpha}_x < 1$, $0 < \widehat{\alpha}_y < 1$, $0 < \widehat{\beta}_x < 1$, $0 < \widehat{\beta}_y < 1$.

We assume that the players employ stationary feedback strategies ("positional controls"), linear in state x_t, i.e.:

$$\begin{cases} u_{it} = e_i \cdot x_t, \ i = 1,\ 2, \\ u_t = (e_1 + e_2) \cdot x_t = e \cdot x_t \end{cases}, \ 0 \leq e < 1, \tag{6}$$

where e_i could be considered as the i-th player's fishing intensity is a share of the current fish stock to be harvested by the player i at each stage.

Lemma 1. (See the proof in [15]) *For each admissible strategy profile meeting (6) the dynamic model (4) implies the following steady-state fish stocks*

$$\begin{cases} x_\infty = (1-e)^{\frac{\widehat{\alpha}_x(1-\widehat{\alpha}_y) + \widehat{\beta}_x\widehat{\beta}_y}{K}}, \\ y_\infty = (1-e)^{\frac{\widehat{\beta}_y}{K}}, \end{cases} \tag{7}$$

where $K = (1 - \widehat{\alpha}_x)(1 - \widehat{\alpha}_y) - \widehat{\beta}_x\widehat{\beta}_y$.

Corollary 1. *It follows from (7) that the steady-state stocks x^∞ and y^∞ are decreasing functions of the total harvested share e.*

In a bicriteria model each player $i = 1, 2$ has a vector objective function $\widetilde{H}_i(\cdot) = \left(H_i^1(\cdot), H_i^2(\cdot)\right)$ and tries to maximize two performance criteria simultaneously. The first performance criterion

$$H_i^1(\cdot) = \sum_{\tau=0}^{\infty} \delta^\tau \cdot \ln u_{i\tau}, \tag{8}$$

has standard economic sense (see, e.g., [1, 3, 4, 6, 13, 14]), where $\delta \in (0, 1)$ is a discount factor. The second (environmental) criterion describes the players' environmental concern (see, e.g., [6, 12, 13, 17, 18]):

$$H_i^2(\cdot) = \sum_{\tau=0}^{\infty} \delta^\tau \cdot \ln (x_\tau \cdot y_\tau). \tag{9}$$

Henceforth, we denote by $G^{1,2}(n = 2, t_0, x_0, y_0)$ the bicriteria infinite-horizon multistage 2-player game starting at $t_0 = 0$ with initial stocks x_0 and y_0, with discrete dynamics (4), feedback information structure (i.e., $u_i(\cdot) = u_i(\tau, x_\tau, y_\tau)$, $\tau = 0, 1, \ldots, \infty; i = 1, 2$) and vector objective functions with components (8) and (9). Similarly, we denote by $G^{1,2}(n = 2, t, x_t, y_t)$ the bicriteria subgame starting at the current time $\tau = t$ and the current state (x_t, y_t). Note that the subgame vector objective functions in $G^{1,2}(n = 2, t, x_t, y_t)$ have the following form:

$$\widetilde{H}_i^t(\cdot) = \left(\sum_{\tau=t}^{\infty} \delta^{\tau-t} \ln u_{i\tau}, \sum_{\tau=t}^{\infty} \delta^{\tau-t} \ln (x_\tau \cdot y_\tau) \right), \ i = 1, 2. \tag{10}$$

As it was proved in [12, 13, 19–21], to derive non-cooperative and cooperative solutions for multicriteria game $G^{1,2}(n = 2, t_0, x_0, y_0)$ one may deal with a corresponding weighted game $G\left(n = 2, t_0, x_0, y_0, (\lambda_i^1, \lambda_i^2)_{i=1,2}\right)$ with the same dynamics (4), but with scalar objective functions of the following form

$$\widehat{H}_i(\cdot) = \lambda_i^1 H_i^1(\cdot) + \lambda_i^2 H_i^2(\cdot), \tag{11}$$

where $\lambda_i^1 \in (0, 1]$ and $\lambda_i^2 \in [0, 1)$ are some weighting coefficients which the player i assigns to her economic and environmental performance criterion respectively, $\lambda_i^1 + \lambda_i^2 = 1$, $i = 1, 2$. Further, to save on the model parameters we divide both parts of (11) on $\lambda_i^1 > 0$ and consider the weighted game $G\left(n = 2, t_0, x_0, y_0, (\lambda_i)_{i=1,2}\right)$ with the normalized objective function

$$H_i(\cdot) = H_i^1(\cdot) + \lambda_i H_i^2(\cdot), \tag{12}$$

where $\lambda_i = \dfrac{1 - \lambda_i^1}{\lambda_i^1}$ could be interpreted as a relative weight of the environmental criterion, $\lambda_i \geq 0$, $i = 1, 2$.

It is worth noting that the higher λ_i, the more the environmental concern of player i. The extreme case when $\lambda_i = 0$ means that the player i aims to maximize only economic performance criterion, while $\lambda_i < 1$ ($\lambda_i > 1$) means that the economic objective is more (less) important for player i than the environmental one.

2. Non-Cooperative Solution

We use a subgame-perfect Pareto equilibrium concept ([12, 13, 19–21]) as a non-cooperative solution of the dynamic game model under consideration. For any $a, b \in R^k$ vector inequality $a \geq b$ means that $a_j \geqslant b_j$ for $j = \overline{1, k}$ and at least one inequality is strict.

Definition 1. [19, 20] *A strategy profile $\overline{u}(\cdot) = (\overline{u}_i(\cdot))_{i=1,2}$ is called a Pareto equilibrium (PE) in multicriteria game $G^{1,2}(n = 2, t_0, x_0, y_0)$ with two players if, for each player $i = 1, 2$, there is no strategy $u_i(\cdot)$ such that*

$$\widetilde{H}_i(u_i(\cdot), \overline{u}_{3-i}(\cdot)) \geq \widetilde{H}_i(\overline{u}(\cdot)).$$

Denote by $PE(G^{1,2}(n=2,t_0,x_0,y_0))$ the set of all Pareto equilibria in $G^{1,2}(n=2,t_0,x_0,y_0)$.

Definition 2. [20, 21] *A strategy profile $\overline{u}(\cdot)$ is called a subgame-perfect Pareto equilibrium (SPPE) in $G^{1,2}(n=2,t_0,x_0,y_0)$ if, for each current time instant $t=0,1,\ldots,\infty$ and each state (x_t,y_t) the restrictions of strategies $\overline{u}_i(\cdot)$, $i=1$ 2, to the subgame $G^{1,2}(n=2,t,x_t,y_t)$ form a Pareto equilibrium in this subgame.*

Each $SPPE$ in $G^{1,2}(n=2,t_0,x_0,y_0)$ can be obtained as a subgame-perfect Nash equilibrium (SPE) [22] of a weighted game $G\,(n=2,t_0,x_0,y_0,(\lambda_i)_{i=1,2})$ for some vector of relative weights $(\lambda_1,\lambda_2)\in R_+^2$ (see [19–21]).

We use the dynamic programming method (see, e.g., [11, 23]) to derive a SPE in feedback strategies for multistage game $G\,(n=2,t_0,x_0,y_0,(\lambda_i)_{i=1,2})$. Let $V_i(t,x,y)$ denote the i-th player's normalized objective function (12) in the subgame $G\,(n=2,t,x,y,(\lambda_i)_{i=1,2})$ starting at time t and state (x,y).

Proposition 1. *A bicriteria game $G^{1,2}(n=2,t_0,x_0,y_0)$ for each pair (λ_1,λ_2), $0\leqslant\lambda_1\leqslant\lambda_2$, possesses a unique subgame perfect Pareto equilibrium in stationary strategies:*

$$\begin{cases} u_1^P(\cdot)=e_1^P\cdot x_t=\gamma_2^P\cdot\left[\frac{\delta}{\Delta}\cdot\gamma_1^P\cdot\gamma_2^P+\gamma_1^P+\gamma_2^P\right]^{-1}\cdot x_t,\\ u_2^P(\cdot)=e_2^P\cdot x_t=\gamma_1^P\cdot\left[\frac{\delta}{\Delta}\cdot\gamma_1^P\cdot\gamma_2^P+\gamma_1^P+\gamma_2^P\right]^{-1}\cdot x_t,\end{cases}\tag{13}$$

where

$$\begin{cases}\Delta=(1-\delta\widehat{\alpha}_x)\cdot(1-\delta\widehat{\alpha}_y)-\delta^2\widehat{\beta}_x\cdot\widehat{\beta}_y,\\ \gamma_i^P=\widehat{\alpha}_x\cdot\Delta_{A_i}^P+\widehat{\beta}_y\cdot\Delta_{B_i}^P,\ i=1,2,\\ \Delta_{A_i}^P=(1+\lambda_i)\cdot(1-\delta\widehat{\alpha}_y)+\lambda_i\cdot\delta\cdot\widehat{\beta}_y,\ i=1,2,\\ \Delta_{B_i}^P=\lambda_i\cdot(1-\delta\widehat{\alpha}_x)+(1+\lambda_i)\cdot\delta\cdot\widehat{\beta}_x,\ i=1,2.\end{cases}\tag{14}$$

Proof. We adopt the following form of the i-th player's value function that represents her $SPPE$ payoff in the subgame $G(n=2,t,x,y,\lambda_1,\lambda_2)$:

$$V_i^P(t,x,y)=A_i^P\ln x+B_i^P\ln y+D_i^P,\ i=1,2.\tag{15}$$

Using a dynamic programming method, the $SPPE$ in feedback strategies is derived by solving the Bellman equations

$$V_i^P(t,x,y)\ =\max_{u_i\geqslant 0}\left\{\ln u_i+\lambda_i\ln(x\cdot y)+\delta\cdot V_i^P(t+1,x_{t+1},y_{t+1})\right\},\ i=1,2.\tag{16}$$

Taking (4), (6) and (15) into account, we can rewrite the system (16) in the following form:

$$\begin{aligned}&A_i^P\ln x+B_i^P\ln y+D_i^P=\max_{e_i\geqslant 0}\Big\{\ln e_i+(1+\lambda_i)\ln x+\lambda_i\ln y+\\&+\delta\cdot\left[A_i^P(\widehat{\alpha}_x\ln x+\widehat{\alpha}_x\ln(1-e_i-e_{3-i})+\widehat{\beta}_x\ln y)+\right.\\&\left.+B_i^P(\widehat{\alpha}_y\ln y+\widehat{\beta}_y\ln x+\widehat{\beta}_y\ln(1-e_i-e_{3-i}))+D_i^P\right]\Big\},\ i=1,2.\end{aligned}\tag{17}$$

Suppose that we have already derived the shares e_i and hence the strategies $u_i=e_i\cdot x, i=1,2$, that maximize the r.h.s. of (17). Then we extract the coefficients for $\ln x$ and $\ln y$ to get the following system for $i=1,2$:

$$\begin{cases}A_i^P=1+\lambda_i+\delta\cdot A_i^P\widehat{\alpha}_x+\delta B_i^P\widehat{\beta}_y,\\ B_i^P=\lambda_i+\delta\cdot A_i^P\widehat{\beta}_x+\delta B_i^P\widehat{\alpha}_y.\end{cases}\tag{18}$$

Note, that all constants Δ, $\Delta_{A_i}^P$ and $\Delta_{B_i}^P$ defined in (14) are positive due to (2). Solving (18) yields to

$$A_i^P=\frac{\Delta_{A_i}^P}{\Delta},B_i^P=\frac{\Delta_{B_i}^P}{\Delta},\ i=1,2.\tag{19}$$

Further, we can differentiate the r.h.s. of (17) w.r.t. e_i:

$$\frac{1}{e_i} + \delta \left[\widehat{\alpha}_x \cdot A_i^P + \widehat{\beta}_y \cdot B_i^P \right] \cdot \frac{(-1)}{1 - e_i - e_{3-i}} = 0, \ i = 1, 2. \tag{20}$$

Rearranging (20) and using notations (14) we obtain

$$e_i \cdot \left(\delta \frac{\gamma_i^P}{\Delta} + 1 \right) + e_{3-i} = 1, \ i = 1, 2. \tag{21}$$

Solving (21) for two asymmetric players ($0 \leqslant \lambda_1 \leqslant \lambda_2$) we obtain the unique interior solution

$$e_i^P = \gamma_{-i}^P \cdot \left[\frac{\delta}{\Delta} \gamma_1^P \gamma_2^P + \gamma_1^P + \gamma_2^P \right]^{-1}, \ i = 1, 2. \tag{22}$$

Note that $e^P = e_1^P + e_2^P = \dfrac{\gamma_1^P + \gamma_2^P}{\frac{\delta}{\Delta} \gamma_1^P \gamma_2^P + \gamma_1^P + \gamma_2^P} < 1$ due to (2) and (14), therefore strategies (13) form an admissible strategy profile meeting (3).

□

3. Cooperative Behavior

Following [12,13,16–18], to find a cooperative solution for each subgame $G^{1,2}(n = 2, t, x_t, y_t)$ with asymmetric players, we assume that players are jointly choose their strategies to maximize the weighted sum of their normalized objective functions $H_i^t(\cdot)$, $i = 1, 2$.

Let $\mu_i > 0$ denote the coefficient of the i-th player's relative importance, $i = 1, 2$. Then the cooperative value function in $G^{1,2}(n = 2, t_0, x_0, y_0)$ takes the form:

$$\begin{aligned} V^C(\cdot) = V_1^C(\cdot) + V_2^C(\cdot) \ &= \mu_1 \sum_{\tau=0}^{\infty} \delta^\tau \left[\ln\left(\theta_1 u(\tau)\right) + \lambda_1 \ln\left(x_\tau y_\tau\right) \right] + \\ &+ \mu_2 \sum_{\tau=0}^{\infty} \delta^\tau \left[\ln\left(\theta_2 u(\tau)\right) + \lambda_2 \ln\left(x_\tau y_\tau\right) \right], \end{aligned} \tag{23}$$

where $u(\tau) = u(\tau, x_\tau, y_\tau)$ denotes the total current harvest of both players, $u_i(\tau, x_\tau, y_\tau) = \theta_i \cdot u(\tau, x_\tau, y_\tau)$, $i = 1, 2$, while $\theta_i > 0$ denotes the exact harvest share of player i under cooperative mode, $\theta_1 + \theta_2 = 1$.

To derive the players' cooperative strategies for given vector $(\mu_1, \mu_2) \in R_+^2$ we have to maximize value function (23) first in $u(\cdot)$ and then in θ_1.

Proposition 2. *A bicriteria two player game $G^{1,2}(n = 2, t_0, x_0, y_0)$ with asymmetric players ($0 \leqslant \lambda_1 \leqslant \lambda_2$) has the unique cooperative solution in stationary feedback strategies for a given vector $(\mu_1, \mu_2) \in R_+^2$:*

$$u^C(\cdot) = e^C \cdot x_t = \frac{\mu_1 + \mu_2}{(\mu_1 + \mu_2) + \delta \left[A^C \cdot \widehat{\alpha}_x + B^C \cdot \widehat{\beta}_y \right]} \cdot x_t, \tag{24}$$

where

$$\begin{cases} A^C = \frac{\Delta_A^C}{\Delta}, B^C = \frac{\Delta_B^C}{\Delta}, \\ \Delta_A^C = (1 - \delta \widehat{\alpha}_y) \cdot \left[\mu_1 (1 + \lambda_1) + \mu_2 (1 + \lambda_2) \right] + \delta \widehat{\beta}_y \cdot \left[\mu_1 \lambda_1 + \mu_2 \lambda_2 \right], \\ \Delta_B^C = (1 - \delta \widehat{\alpha}_x) \cdot \left[\mu_1 \lambda_1 + \mu_2 \lambda_2 \right] + \delta \widehat{\beta}_x \cdot \left[\mu_1 (1 + \lambda_1) + \mu_2 (1 + \lambda_2) \right]. \end{cases} \tag{25}$$

The i-th player's cooperative strategy is given by

$$u_i^C(\tau, x_\tau, y_\tau) = e_i^C \cdot x_\tau = \frac{\mu_i}{(\mu_1 + \mu_2) + \delta \left[A^C \cdot \widehat{\alpha}_x + B^C \cdot \widehat{\beta}_y \right]} \cdot x_\tau, \ i = 1, 2. \tag{26}$$

Proof. Again, we use a suitable functional form for the cooperative value function (23), namely:

$$V^C(t,x,y) = A^C \cdot \ln x + B^C \ln y + D^C. \tag{27}$$

Applying the dynamic programming method to the value function (23), (27) yields the Bellman equation:

$$\begin{gathered} A^C \cdot \ln x + B^C \ln y + D^C = \max_{e\geqslant 0} \Big\{ \mu_1 \big[(\ln\theta_1 + \ln e + \ln x) + \lambda_1 \ln x + \lambda_1 \ln y\big] + \\ + \mu_2 \big[(\ln\theta_2 + \ln e + \ln x) + \lambda_2 \ln x + \lambda_2 \ln y\big] + \delta \Big[A^C(\widehat{\alpha}_x \ln x + \widehat{\alpha}_x \ln(1-e) + \widehat{\beta}_x \ln y) + \\ + B^C(\widehat{\beta}_y \ln x + \widehat{\beta}_y \ln(1-e) + \widehat{\alpha}_y \ln y) + D^C \Big] \Big\}. \end{gathered} \tag{28}$$

For the optimal total share e^C to be harvested at each stage which solves (28) we extract the coefficients for $\ln x$ and $\ln y$ in the r.h.s. and the l.h.s. of (28). Then the following system of equations holds:

$$\begin{cases} A^C = \mu_1(1+\lambda_1) + \mu_2(1+\lambda_2) + \delta\widehat{\alpha}_x \cdot A^C + \delta\widehat{\beta}_y \cdot B^C, \\ B^C = \mu_1\lambda_1 + \mu_2\lambda_2 + \delta\widehat{\beta}_x \cdot A^C + \delta\widehat{\alpha}_y \cdot B^C. \end{cases} \tag{29}$$

Using constants Δ_A^C, Δ_B^C, defined in (25) and Δ defined in (14), we derive the unique solution of (29) in the following form:

$$A^C = \frac{\Delta_A^C}{\Delta}, \ B^C = \frac{\Delta_B^C}{\Delta}.$$

Note that $A^C > 0$, $B^C > 0$ due to assumptions (2).

Using the first order condition for the r.h.s. of (28) yields:

$$\frac{\mu_1+\mu_2}{e} + \delta \cdot \left[(A^C \cdot \widehat{\alpha}_x + B^C \cdot \widehat{\beta}_y) \cdot \frac{(-1)}{1-e} \right] = 0.$$

Solving the latter equation one can get the interior solution

$$e^C = \frac{(\mu_1+\mu_2)}{(\mu_1+\mu_2) + \delta[A^C \cdot \widehat{\alpha}_x + B^C \cdot \widehat{\beta}_y]}, \tag{30}$$

hence (24) holds. Note that the cooperative stage harvest e^C is admissible since $e^C < 1$, and does not depend on θ_1 and θ_2.

Further, to derive coefficients θ_1 and θ_2 in (23) we have the following constrained optimization problem:

$$\begin{cases} \max\limits_{\theta_1,\theta_2} (\mu_1 \ln\theta_1 + \mu_2 \ln\theta_2), \\ \theta_1 + \theta_2 = 1. \end{cases} \tag{31}$$

Solving (31) yields $\theta_i = \dfrac{\mu_i}{\mu_1+\mu_2}$, $i = 1,2$, and hence formulae (26) hold.

□

4. Numerical Simulation for Different Levels of the Players' Asymmetry

In this section, we mainly focus on the problem how the level of the players' environmental concern may affect the solutions properties. For numerical simulation we use the following parameters values as a baseline scenario, which is close to one explored in [4, 15]: $\alpha_x = 0,6$; $\alpha_y = 0,5$; $\beta_x = 0,1$; $\beta_y = 0,2$; $x_0 = 0,1$; $y_0 = 0,1$; $s = 0,0185$.

To represent the different levels of the player's environmental concern we employ the following patterns:

- pattern 1 (SYMM) represents the case of absolutly symmetric players with 20 percent level of their environmental concern, namely $\lambda_i^1 = 0,8$, $\lambda_i^2 = 0,2$ and $\lambda_i = 0,25$ for $i = 1,2$;

- pattern 2 (LA – Low Asymmetry) implies a low level of the players asymmetry, namely $\lambda_1^1 = 0,9$, $\lambda_1^2 = 0,1$, hence $\lambda_1 = 1/9 = 0,1111(1)$ and $\lambda_2^1 = 0,7$, $\lambda_2^2 = 0,3$, hence $\lambda_2 = 3/7 = 0,42857\ldots$;
- pattern 3 (HA – High Asymmetry) reflects an (extremely) high level of the players asymmetry, namely $\lambda_1^1 = 1$, $\lambda_1^2 = 0$, hence $\lambda_1 = 0$ and $\lambda_2^1 = 0,6$, $\lambda_2^2 = 0,4$, hence $\lambda_2 = 2/3 = 0,6666(6)$.

Note that all three patterns above imply the same average level of the players' environmental concern, i.e.: $(\lambda_1^2 + \lambda_2^2) \cdot 1/2 = 0,2$.

First, we run the model for $t \in [0,30]$ in series for three patterns chosen above to compare stocks evolution in cooperative and non-cooperative scenario. It turns out that the cooperative mode is better for the environment (the stocks x_t^C are higher than x_t^P in the long run) regardless of the level of the players' asymmetry and of the initial conditions. A similar remark is true for the stocks y_t^C and y_t^P in MPA as well as for the steady-state stocks. Moreover, taking Prop. 1, Prop. 2 and Cor. 1 into account one can predict how the level of the players' asymmetry affects the total harvested share e. A typical time graphs of the fish stocks evolution are presented in Fig. 1 and Fig. 2 (for the pattern LA).

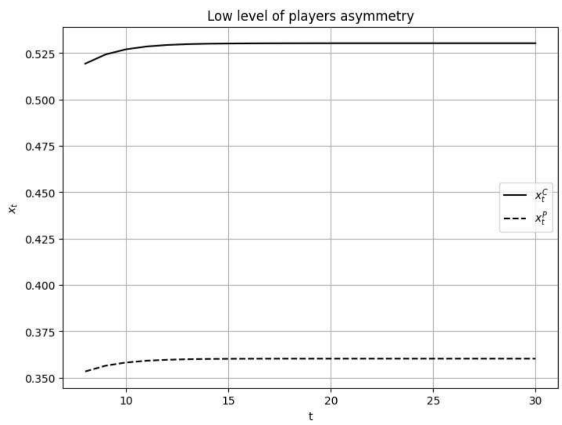

Fig. 1. x_t^C (solid line) and x_t^P (dashed line) for players with low level of asymmetry

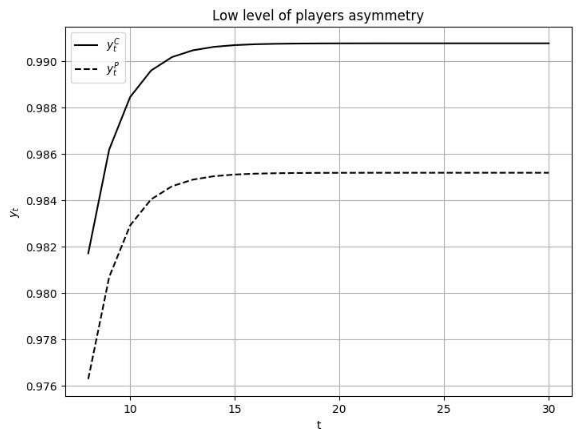

Fig. 2. y_t^C (solid line) and y_t^P (dashed line) for players with low level of asymmetry

Next, to evaluate how the size s of the fish bank affects the current stock x_t, we run the model for $t \in [0,30]$ for different levels of the players' asymmetry. Following [4,15] we adopt the following benchmark levels of s:

- $s = 0,0185$ – today approximated size of the fish bank zone;
- $s = 0,1$ – today approximated size of the marine protected areas, where fishing is either restricted or completely prohibited by law;
- $s = 0,2$ – probable future size which is consistent with modern trends and conservation goals (see [4,9]).

The results of numerical simulation are presented in Fig. 3 – 5 for cooperative scenario. Three interesting observations can be deduced from these findings. First, an increasing of the fish bank size could lead to the decreasing of the stock x_t^C available for fishery in the long run. Second, the higher the level of the players' asymmetry, the weaker the mentioned impact of the MPA size on the fish stock (at least, for the considered range of the parameters values). Third, higher level of the players' asymmetry could enlarge the current fish stock x_t^C for given size s of the fish bank.

Concluding Remarks

The introduced multi-objective fish war dynamic game model with fish bank treats the players asymmetry as the different levels of the players' environmental concern. This bicriteria

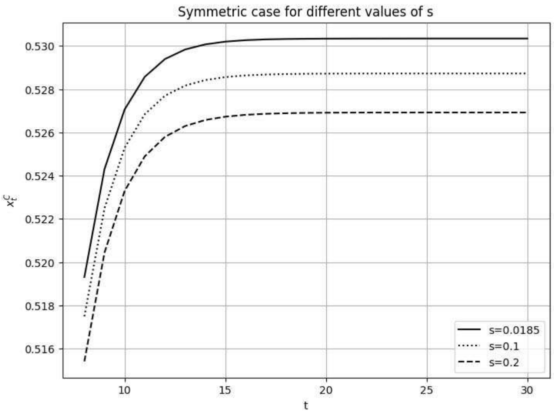

Fig. 3. Stock x_t^C evolution for different s (SYMM, solid line corresponds to $s = 0,0185$, dotted line corresponds to $s = 0,1$, dashed line corresponds to $s = 0,2$)

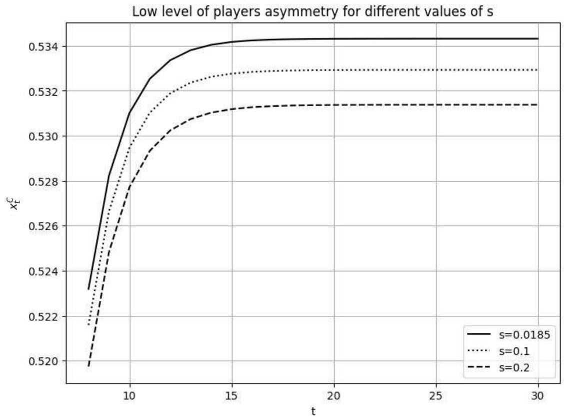

Fig. 4. Stock x_t^C evolution for different s (LA, solid line corresponds to $s = 0,0185$, dotted line corresponds to $s = 0,1$, dashed line corresponds to $s = 0,2$)

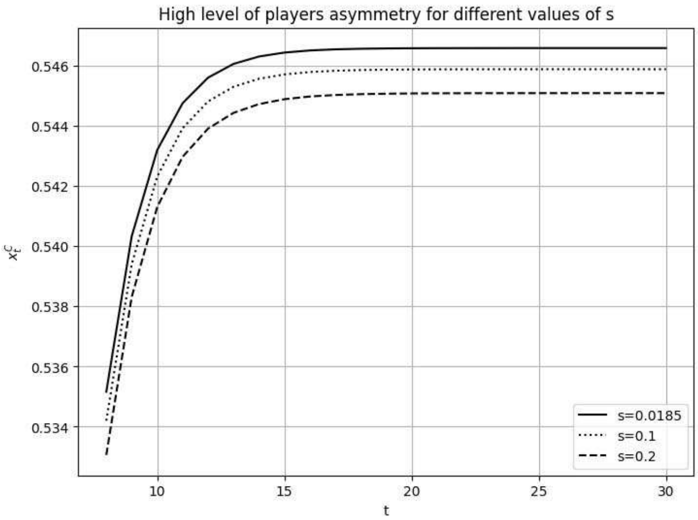

Fig. 5. Stock x_t^C evolution for different s (HA, solid line corresponds to $s = 0,0185$, dotted line corresponds to $s = 0,1$, dashed line corresponds to $s = 0,2$)

asymmetric model retains two good properties of the standard infinite-horizon model with only economic objective and symmetric players. Namely, both non-cooperative and cooperative solutions are derived analytically in feedback stationary strategies (Prop. 1 and Prop. 2), and the cooperative regime of the resource extraction is better for the environment than the non-cooperative scenario (see Sect. 4).

In addition, the bicriteria model with two asymmetric players enables us to evaluate how the level of the players' asymmetry may affect the solutions properties. Theoretical results (Prop. 1, 2) and the numerical simulation findings could be useful when designing sustainable long-run fishery and evaluating different environmental preservation polices.

Note two implications for the the fishery management policy elaboration. First, the results obtained could be applied to forecast and evaluate the consequences of possible MPA expansion taking into account the different values of the players' environmental concern.

Second, if the relative weight of the environmental criterion of some player is estimated as too small, a regulator may consider an option to design and implement some efforts to increase the level of that player's environmental concern before the MPA expansion. These efforts may include such administrative-legal solutions as logistic advantages, subsidies, tax incentives, and other benefits for those with higher level of environmental valuation, accompanied by creating some administrative barriers for fishing companies with low level of the environmental concern.

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БИКРИТЕРИАЛЬНАЯ ДИНАМИЧЕСКАЯ ИГРА «РЫБНЫХ ВОЙН» С АСИММЕТРИЧНЫМИ ИГРОКАМИ

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Рассматриваем динамическую многокритериальную модель управления рыболовством, учитывающую процесс миграции ресурса между зоной свободного вылова и природоохранной зоной, где промысел запрещен. Предложенное обобщение стандартной однокритериальной игровой модели подразумевает, что каждый игрок стремится одновременно максимизировать два критерия: экономическую выгоду и экологический критерий (поддержание размеров популяции в каждой зоне). В статье изучается игра с двумя игроками, имеющими различную оценку значимости экологического критерия. Используя метод динамического программирования, мы строим аналитически некооперативное и кооперативное решение в стационарных позиционных стратегиях для бесконечношаговой бикритериальной игры с двумя асимметричными игроками. Для изучения отдельных свойств решений проводится численное моделирование.

Ключевые слова: добыча возобновляемых ресурсов; многокритериальная игра; динамические игры; равновесие по Парето; кооперативное решение; рыболовство.

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