

STOCHASTIC LEONTIEFF-TYPE EQUATIONS WITH MULTIPLICATIVE EFFECT IN SPACES OF COMPLEX-VALUED "NOISES"

A. L. Shestakov, South Ural State University, Chelyabinsk, Russian Federation,
admin@susu.ac.ru,

M. A. Sagadeeva, South Ural State University, Chelyabinsk, Russian Federation,
sam79@74.ru

We consider a Leontieff-type stochastic equation, that is, a system of differential equations implicit with respect to the time derivative in the spaces of random processes. The concepts previously introduced for the spaces of differentiable "noise" using the Nelson–Gliklikh derivative carry over to the case of complex-valued "noise"; in addition, the right-hand side of the equation is subject to multiplicative effect of a special form. We construct a solution to the Showalter–Sidorov problem for Leontieff-type equations with multiplicative effect of a complex-valued process of special form. Aside from the introduction and references, the article consists of two parts. In the first part we carry over various concepts of the space of real-valued differentiable "noise" to the complex-valued case. In the second part we construct a Showalter–Sidorov solution to a Leontieff-type equation with multiplicative effect of a complex-valued process of special form. The list of references is not intended to be complete and reflects only the authors' personal preferences.

Keywords: *Leontieff-type equations; multiplicative effect; Wiener process; Nelson–Gliklikh derivative; space of complex-valued "noises"; "white noise".*

Introduction

In the spaces of random processes, consider the stochastic equation

$$L \overset{\circ}{\xi}(t) = e^{i\alpha t} M \xi(t) + N \overset{\circ}{W}_k(t), \quad \ker L \neq \{0\}, \quad (1)$$

where α is a real-valued Gaussian random variable, L , M , and N are matrices of complex coefficients, $\overset{\circ}{\xi}(t)$ is the required complex-valued random process with the Nelson–Gliklikh derivative $\overset{\circ}{\xi}$, and $W_k(t)$ is the k -Wiener process.

Ito pioneered the study of linear ordinary stochastic differential equations, and later Stratonovich and Skorokhod joined in. The approach of Ito–Stratonovich–Skorokhod is still popular [1, 2] in the finite-dimensional case. Moreover, it was successfully extended to an infinite-dimensional situation [3] and even to Sobolev-type equations [4, 5, 6]. The right-hand side involved the differential of a Wiener processes, whose generalized derivative is traditionally treated as white noise. Let us also mention the approach of Mel'nikova's school [7], in which ordinary stochastic differential equations are considered in Schwartz spaces, where the generalized derivative of a Wiener process makes sense.

A new approach in stochastic ordinary differential equations arose [8] and is actively developing [9], in which "white noise" is regarded as the Nelson–Gliklikh derivative of a Wiener processes. This "white noise" is more adequate for the Einstein–Smoluchowski theory of Brownian motion than the traditional white noise [8, 9]. Originally "white noise"

was used in the theory of optimal measurements [10, 11], which required constructing a special space of real-valued "noises" [9]. In this article for the finite-dimensional case we carry over the concepts and results of [6, 9] to complex-valued random processes. In these spaces we construct a solution to a Leontieff-type equation (1) with multiplicative effect of a complex-valued process of special form basing on the deterministic case [11, 12].

1. Spaces of Complex-Valued "Noises"

Take a complete probability space $\Omega \equiv (\Omega, \mathcal{A}, \mathbf{P})$ and the set \mathbb{C} of complex numbers equipped with the structure of a Borel σ -algebra. A measurable mapping $\xi : \Omega \rightarrow \mathbb{C}$ is called a *random variable*. The set of random variables constitutes a Hilbert space with the inner product $(\xi_1, \xi_2) = \mathbf{E}\xi_1\overline{\xi_2}$. Denote this Hilbert space by \mathbf{L}_2 . Below the random variables $\xi \in \mathbf{L}_2$ with the normal (Gaussian) distribution are important, and we refer to them as *Gaussian variables*.

Consider now a σ -subalgebra \mathcal{A}_0 of the σ -algebra \mathcal{A} . Construct the space \mathbf{L}_2^0 of random variables which are measurable with respect to \mathcal{A}_0 . It turns out that \mathbf{L}_2^0 is a subspace of \mathbf{L}_2 . Denote by $\Pi : \mathbf{L}_2 \rightarrow \mathbf{L}_2^0$ the corresponding orthogonal projection. For $\xi \in \mathbf{L}_2$ refer to $\Pi\xi$ as the *conditional expectation* of the random variable ξ and denote it by $\mathbf{E}(\xi|\mathcal{A}_0)$. It is not difficult to observe that $\mathbf{E}(\xi|\mathcal{A}_0) = \mathbf{E}\xi$ for $\mathcal{A}_0 = \{\emptyset, \Omega\}$ and $\mathbf{E}(\xi|\mathcal{A}_0) = \xi$ for $\mathcal{A}_0 = \mathcal{A}$. Finally, recall that the minimal σ -subalgebra $\mathcal{A}_0 \subset \mathcal{A}$ with respect to which a random variable ξ is measurable is called the σ -algebra *generated by* ξ .

Consider now some interval $\mathfrak{J} \subset \mathbb{R}$ and two mappings. The first mapping $f : \mathfrak{J} \rightarrow \mathbf{L}_2$ associates to each $t \in \mathfrak{J}$ a random variable $\xi \in \mathbf{L}_2$. The second mapping $g : \mathbf{L}_2 \times \Omega \rightarrow \mathbb{C}$ associates to each pair (ξ, ω) a point $\xi(\omega) \in \mathbb{C}$. Refer to a mapping $\eta : \mathfrak{J} \times \Omega \rightarrow \mathbb{C}$ defined as $\eta = \eta(t, \omega) = g(f(t), \omega)$ as a *(one-dimensional) complex-valued random process*. Therefore, for each fixed $t \in \mathfrak{J}$ the random process $\eta = \eta(t, \cdot)$ is a complex random variable, that is, $\eta(t, \cdot) \in \mathbf{L}_2$, while for each fixed $\omega \in \Omega$ the random process $\eta = \eta(\cdot, \omega)$ is called a *(choice of) trajectory*. A random process η is called *continuous* whenever almost surely all its trajectories are continuous (that is, for almost all $\omega \in \Omega$ the trajectories $\eta(\cdot, \omega)$ are continuous). The set of continuous random processes constitutes a Banach space, which we denote by $\mathbf{C}(\mathfrak{J}; \mathbb{C})$. A continuous random process with (independent) Gaussian random variables is called a *Gaussian process*.

The most important example of a continuous complex-valued Gaussian random process is the (one-dimensional) Wiener process $\beta = \beta(t)$ modeling the Brownian motion on a complex plane in the Einstein–Smoluchowski theory. It enjoys the following properties:

(W1) almost surely $\beta(0) = 0$, almost surely all its trajectories $\beta(t)$ are continuous, and for all $t \in \overline{\mathbb{R}}_+ (= \{0\} \cup \mathbb{R})$ the random variable $\beta(t)$ is Gaussian;

(W2) the expectation is $\mathbf{E}(\beta(t)) = 0$ and the autocorrelation function is $\mathbf{E}(|\beta(t) - \beta(s)|^2) = |t - s|$ for all $s, t \in \overline{\mathbb{R}}_+$.

Remark 1. For the process with properties (W1), (W2) also satirsfire

(W3) the trajectories $\beta(t)$ are nondifferentiable at every point $t \in \overline{\mathbb{R}}_+$ and of unbounded variation on every arbitrarily small interval.

A random process β with properties (W1)–(W3) is called *Brownian motion*.

Fixing $\eta \in \mathbf{C}(\mathfrak{J}; \mathbb{C})$ and $t \in \mathfrak{J} (= (\varepsilon, \tau) \subset \mathbb{R})$, denote by \mathcal{N}_t^η the σ -algebra generated by the random variable $\eta(t)$. Put $\mathbf{E}_t^\eta = \mathbf{E}(\cdot|\mathcal{N}_t^\eta)$ for brevity.

Definition 1. On assuming that $\eta \in \mathbf{C}(\varepsilon, \tau; \mathbb{C})$, refer as the (right left) derivative in the mean $D\eta(t, \cdot)$ ($D_*\eta(t, \cdot)$) of the random process η at $t \in (\varepsilon, \tau)$ to the random variable

$$D\eta(t, \cdot) = \lim_{\Delta t \rightarrow 0+} E_t^\eta \left(\frac{\eta(t + \Delta t, \cdot) - \eta(t, \cdot)}{\Delta t} \right) \quad \left(D_*\eta(t, \cdot) = \lim_{\Delta t \rightarrow 0+} E_t^\eta \left(\frac{\eta(t, \cdot) - \eta(t - \Delta t, \cdot)}{\Delta t} \right) \right)$$

whenever the limit exists in the sense of uniform metric on \mathbb{R} . A random process η is called (right left) differentiable in the mean on (ε, τ) whenever at each point $t \in (\varepsilon, \tau)$ its right (left) derivative in the mean exists.

Therefore, take a random process $\eta \in \mathbf{C}(\varepsilon, \tau; \mathbb{C})$ which is right (left) differentiable in the mean on (ε, τ) . Its right (left) derivative in the mean is a random process as well, which we denote by $D\eta$ ($D_*\eta$). If $\eta \in \mathbf{C}(\varepsilon, \tau; \mathbb{C})$ is both right and left differentiable in the mean on (ε, τ) then we can define the symmetric (antisymmetric) derivative in the mean $D_S\eta = \frac{1}{2}(D + D_*)\eta$ ($D_A\eta = \frac{1}{2}(D_* - D)\eta$). Since Nelson introduced derivatives in the mean [13] and Gliklikh developed their theory [2], for brevity we call the symmetric derivative in the mean D_S of a random process η the Nelson–Gliklikh derivative and denote it by $\overset{\circ}{\eta}$, that is, put $D_S\eta \equiv \overset{\circ}{\eta}$. For $\ell \in \mathbb{N}$, denote by $\overset{\circ}{\eta}^{(\ell)}$ the order ℓ Nelson–Gliklikh derivative of η . Observe that if the trajectories of η are almost surely continuously differentiable in the ordinary sense on (ε, τ) then their Nelson–Gliklikh derivative coincides with the ordinary derivative. For instance, this is so for the random real-valued process $\eta = \alpha \sin(\nu t)$, where α is a Gaussian random real-variable and $\nu \in \mathbb{R}_+$ is a fixed constant, while $t \in \mathbb{R}$ has the meaning of time.

Theorem 1. (Gliklikh) We have $\overset{\circ}{\eta}^{(\ell)}(t) = (-1)^{\ell+1}(2t)^{-\ell}\eta(t)$ for all $t \in \mathbb{R}_+$ and $\ell \in \mathbb{N}$.

Consider the space $\mathbf{C}^\ell(\mathcal{J}; \mathbb{C})$, with $\ell \in \mathbb{N}$, of random process in $\mathbf{C}(\mathcal{J}; \mathbb{C})$ whose trajectories are almost surely Nelson–Gliklikh differentiable on \mathcal{J} through order ℓ . If $\mathcal{J} \subset \mathbb{R}_+$ then Theorem 1 implies the existence of the derivative $\overset{\circ}{\beta} \in \mathbf{C}^1(\mathcal{J}; \mathbb{C})$, which we call a (one-dimensional) "white noise". In [9] the spaces $\mathbf{C}^1(\mathcal{J}; \mathbb{C})$ are called the spaces of differentiable "noises".

Fixing now $k \in \mathbb{N}$, take k independent random processes $\{\eta_1(t), \eta_2(t), \dots, \eta_k(t)\}$ and define the k -dimensional random process (or briefly, a k -random process) as $\Theta(t) = \sum_{j=1}^k \eta_j(t)e_j$, where e_j for $j = \overline{1, k}$ are the standard basis vectors in \mathbb{C}^k . It is obvious that almost surely all its trajectories are continuous whenever $\eta_j \in \mathbf{C}(\mathcal{J}; \mathbb{C})$ for $j = \overline{1, k}$ and Nelson–Gliklikh continuously differentiable through order ℓ whenever $\eta_j \in \mathbf{C}^\ell(\mathcal{J}; \mathbb{C})$ for $j = \overline{1, k}$. By analogy with the above, introduce the spaces $\mathbf{C}(\mathcal{J}; \mathbb{C}^k)$ and $\mathbf{C}^\ell(\mathcal{J}; \mathbb{C}^k)$ of continuous and continuously differentiable k -dimensional "noises". As an example, consider the k -dimensional Wiener process (k -Wiener process)

$$W_k(t) = \sum_{j=1}^k \beta_j(t)e_j, \tag{2}$$

where β_j for $j = \overline{1, k}$ are independent Brownian motions. Theorem 1 yields

It follows from (2) that the k -Wiener process W_k enjoys properties (W1)–(W3) with β replaced by W_k . On assuming this replacement made, we have

Theorem 2. For each $k \in \mathbb{N}$ with probability 1 there exists a unique k -Wiener process W_k with properties (W1)–(W3); furthermore, we can express it as (2).

2. Stochastic Leontieff-Type Equations with Multiplicative Effect

Take two square matrices L and M of size k with complex entries. Following [6] and [10], refer to the sets $\rho^L(M) = \{\mu \in \mathbb{C} : \det(\mu L - M) \neq 0\}$ and $\sigma^L(M) = \mathbb{C} \setminus \rho^L(M)$ respectively as the L -resolvent set and L -spectrum of M . It is not difficult to show that either $\rho^L(M) = \emptyset$ or the L -spectrum of M amounts to a finite set of points. In addition, observe that $\rho^L(M)$ and $\sigma^L(M)$ are preserved under the changes of basis. Define also the matrix-valued functions $(\mu L - M)^{-1}$, $R_\mu^L(M) = (\mu L - M)^{-1}L$, and $L_\mu^L(M) = L(\mu L - M)^{-1}$ on $\rho^L(M)$ and call them respectively the L -resolvent, the right and left L -resolvents of M .

Definition 2. For $p \in \{0\} \cup \mathbb{N}$, call M an (L, p) -regular matrix whenever $\rho^L(M) \neq \emptyset$ and ∞ is either a removable singular point (for $p = 0$) or a pole of order $p \in \mathbb{N}$ of the L -resolution of M .

Take an (L, p) -regular matrix M with $p \in \{0\} \cup \mathbb{N}$. Consider the system of Leontieff-type equations

$$L \overset{\circ}{\xi}(t) = e^{i\alpha t} M \xi(t) + N w(t), \quad \ker L \neq \{0\}, \quad (3)$$

where α is a real-valued Gaussian random variable, $\xi(t)$ is the required random process with the Nelson–Gliklikh derivative $\overset{\circ}{\xi}$, and $w = w(t)$ is a random process corresponding to the outside effect.

Impose for (3) the Showalter–Sidorov initial condition

$$[R_\nu^L(M)]^{p+1} (\xi(0) - \xi_0) = 0, \quad \nu \in \rho^L(M). \quad (4)$$

Observe that for the Leontieff-type system [10] this condition is more natural than the traditional Cauchy condition $\xi(0) = \xi_0$.

Take now $\mathfrak{J} = [0, \tau)$. Call a random process $\xi \in \mathbf{C}^1(0, \tau; \mathbb{C}^k)$ a (classical) solution to (3) whenever almost surely all its trajectories satisfy (3) for some random process $w \in \mathbf{C}(0, \tau; \mathbb{C}^k)$, a complex matrix N of size k , and all $t \in (0, \tau)$. Call a solution $\xi = \xi(t)$ to (3) a (classical) solution to problem (3), (4) whenever condition (4) holds as well.

Theorem 3. Take an (L, p) -regular matrix M for $p \in \{0\} \cup \mathbb{N}$ with $\det M \neq 0$. Given a matrix $N : \mathbb{C}^k \rightarrow \mathbb{C}^k$, a random process $w = w(t)$ with

$$(\mathbb{I} - Q)Nw \in \mathbf{C}^{p+1}(\mathfrak{J}; \mathbb{C}^k) \quad \text{and} \quad QNw \in \mathbf{C}(\mathfrak{J}; \mathbb{C}^k), \quad (5)$$

and a random variable $\xi_0 \in \mathbf{L}_2$ independent of w , there exists a unique solution $\xi \in \mathbf{C}^1(\mathfrak{J}; \mathbb{C}^k)$ to problem (3), (4) of the form

$$\begin{aligned} \xi(t) = & \lim_{n \rightarrow \infty} \left[\left(\left(L + \frac{i}{n\alpha} M (e^{i\alpha t} - 1) \right)^{-1} L \right)^n \xi_{0+} \right. \\ & \left. + \int_0^t \left(\left(L + \frac{i}{n\alpha} M (e^{i\alpha t} - e^{i\alpha s}) \right)^{-1} L \right)^n \left(L - \frac{1}{n} M \right)^{-1} (nL_n^L(M))^p Nw(s) ds + \right. \end{aligned}$$

$$+ \sum_{q=0}^p \left(M^{-1} \left(\mathbb{I}_k - (nL_n^L(M))^{p+1} \right) L \right)^q M^{-1} \left((nL_n^L(M))^{p+1} - \mathbb{I}_k \right) (e^{-i\alpha t} D_S)^q (e^{-i\alpha t} Nw(t)) \Big].$$

Proof of this theorem repeats almost verbatim the arguments of [11, 12], taking into account the features of the space of "noises" [6, 9], and is therefore omitted.

However, the "white noise" $w(t) = \overset{\circ}{W}_k(t) = (2t)^{-1}W_k(t)$ doesn't satisfy (5) and we can't substitute it into (3) and (4). So for equation (1) we consider the *weakened Showalter-Sidorov initial condition*

$$\lim_{t \rightarrow 0+} [R_\nu^L(M)]^{p+1} (\xi(t) - \xi_0) = 0, \quad \nu \in \rho^L(M). \quad (6)$$

Using the results from [6] and Theorem 3, we obtain next

Corollary 1. *Take an (L, p) -regular matrix M for $p \in \{0\} \cup \mathbb{N}$ with $\det M \neq 0$. Given a matrix $N : \mathbb{C}^k \rightarrow \mathbb{C}^k$ and a random variable $\xi_0 \in \mathbf{L}_2$ independent of k -Wiener process W_k , there exists a unique solution $\xi \in \mathbf{C}^1(\mathfrak{J}; \mathbb{C}^k)$ to problem (1), (6) of the form*

$$\begin{aligned} \xi(t) = & \lim_{n \rightarrow \infty} \left[\left(\left(L + \frac{i}{n\alpha} M (e^{i\alpha t} - 1) \right)^{-1} L \right)^n \xi_0 + \left(L - \frac{1}{n} M \right)^{-1} \left[(nL_n^L(M))^p N W_k(t) - \right. \right. \\ & \left. \left. - M \int_0^t \left(\left(L + \frac{i}{n\alpha} M (e^{i\alpha t} - e^{i\alpha s}) \right)^{-1} L \right)^n \left(L - \frac{1}{n} M \right)^{-1} (nL_n^L(M))^p N W_k(s) ds \right] + \right. \\ & \left. + \sum_{q=0}^p \left(M^{-1} \left(\mathbb{I}_k - (nL_n^L(M))^{p+1} \right) L \right)^q M^{-1} \left((nL_n^L(M))^{p+1} - \mathbb{I}_k \right) (e^{-i\alpha t} D_S)^q (e^{-i\alpha t} N W_k(t)) \right]. \end{aligned}$$

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СТОХАСТИЧЕСКИЕ УРАВНЕНИЯ ЛЕОНТЬЕВСКОГО ТИПА С МУЛЬТИПЛИКАТИВНЫМ ВОЗДЕЙСТВИЕМ В ПРОСТРАНСТВАХ КОМПЛЕКСНОЗНАЧНЫХ «ШУМОВ»

А.Л. Шестаков, М.А. Сагадеева

В статье рассматривается стохастическое уравнение леонтьевского типа, т.е. система дифференциальных уравнений, неразрешенная относительно производной по времени, в пространствах случайных процессов. При этом введенные ранее с помощью производной Нельсона–Гликлиха понятия для пространств дифференцируемых «шумов», переносятся на случай комплекснозначных «шумов», и, кроме того, в уравнении присутствует мультипликативное воздействие специального вида на правую часть уравнения. В статье строится решение задачи Шоултера–Сидорова для уравнения леонтьевского типа с мультипликативным воздействием комплекснозначного процесса специального вида.

Статья кроме введения и списка литературы содержит две части. В первой из них производится перенос понятий пространства дифференцируемых «шумов» с действительногозначного случая на комплекснозначный, а во второй — строится решение Шоултера–Сидорова для уравнения леонтьевского типа с мультипликативным воздействием комплекснозначного процесса специального вида. Список литературы не претендует на полноту, и отражает лишь личные пристрастия авторов.

Ключевые слова: уравнение леонтьевского типа; мультипликативное воздействие; винеровский процесс; производная Нельсона – Гликлиха; пространство комплекснозначных «шумов»; «белый шум».

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Александр Леонидович Шестаков, доктор технических наук, профессор, кафедра «Информационно-измерительная техника», Южно-Уральский государственный университет (г. Челябинск, Российская Федерация), admin@susu.ac.ru

Минзиля Алмасовна Сагадеева, кандидат физико-математических наук, доцент, кафедра «Математическое моделирование», Южно-Уральский государственный университет (г. Челябинск, Российская Федерация), sam79@74.ru

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