describing the ion-acoustic waves in a plasma in a magnetic field, and the negative values of the parameter λ do not contradict the physical meaning of this problem. Stochastic mathematical model of ion-acoustic waves in a plasma was considered in [3].

1. Analytical Study of the Mathematical Model of Ion-Acoustic Waves in a Plasma in a Magnetic Field. Introduce the eigenfunctions of the Laplace operator Δ in the domain Ω satisfying conditions (3): $\varphi_{kmn} = \left\{ \sin \frac{kx_1}{a} \sin \frac{mx_2}{b} \sin \frac{nx_3}{c} \right\}$, where k, m, $n \in \mathbb{N}$, and the eigenvalues $\lambda_{kmn} = -(k^2 + m^2 + n^2)$. Obviously, the spectrum $\sigma(\Delta)$ is negative, discrete with finite multipicities and thickens only to $-\infty$. Since $\{\varphi_k\} \subset C^{\infty}(\Omega)$, then

$$\mu^4 A - \mu^3 B_3 - \mu^2 B_2 - \mu B_1 - B_0 =$$

$$\sum_{k,m,n=1}^{\infty} [(\lambda_{kmn} - \lambda)\mu^4 + (\lambda_{kmn} - \lambda')\mu^2 - \alpha(\frac{\pi n}{c})^2] < \varphi_{kmn}, \cdot > \varphi_{kmn},$$

where $< \cdot, \cdot >$ is a scalar product in $L^2(\Omega)$.

Lemma 1. [4] *(i) Let* $\lambda \notin \sigma(\Delta)$. *Then the pencil* $\vec{B}$ *is polynomially* $(A, 0)$*-bounded.*

(ii) $(\lambda \in \sigma(\Delta)) \wedge (\lambda \neq \lambda')$. *Then the pencil* $\vec{B}$ *is polynomially* $(A, 1)$*-bounded.*

(iii) $(\lambda \in \sigma(\Delta)) \wedge (\lambda = \lambda')$. *Then the pencil* $\vec{B}$ *is polynomially* $(A, 3)$*-bounded.*

Theorem 1. [4] *(i) Let* $\lambda \notin \sigma(\Delta)$. *Then, for arbitrary* $v_0, v_1, v_2, v_3 \in \mathfrak{U}$ *there exists a unique solution of problem (2) – (4).*

(ii) Let $\lambda \in \sigma(\Delta)$ *и* $\lambda = \lambda'$. *Then for arbitrary* $v_0, v_1, v_2, v_3 \in \mathfrak{U}^1$, *i.e., such that*

$$\sum_{\lambda_{kmn}=\lambda} < \varphi_{kmn}, v_j >= 0, \ j = 0, ..., 3,$$

there exists a unique solution of problem (2) – (4).

2. Numerical Solution Algorithm. Based on the theoretical results there was developed an algorithm for numerical solution of problem (2) – (4) modelling ion-acoustic waves in a plasma in an external magnetic field, implemented in a software environment Maple 15.0. The program uses a phase space method and a modified Galerkin method.

A numerical solution algorithm is shown in a block diagram in picture 1. The developed program allows you to:

1. Specify the sizes of the domain Ω for the mathematical model of ion-acoustic waves in a plasma in an external magnetic field.

2. Enter the parameters of the equation: $\lambda, \lambda', \alpha$; initial data: $v_0(x, y, z), v_1(x, y, z), v_2(x, y, z), v_3(x, y, z)$, and the order of Galerkin approximations N.

3. Print the numerical solution of the problem.

4. Get a graphical image of the received waves with animated distribution over time.

A detailed description of the algorithm (each block of the algorithm corresponds to one step):

Step 1. After the start of the program the number of terms in a Galerkin sum N, parameters λ, λ_1, α, initial data v_0, v_1, v_2, v_3, the positive numbers a, b, c and period $\tau : t \in [0, \tau]$ are entered.

Step 2. In a cycle approximate solution V is represented as the Galerkin sum $\sum\limits_{i,j,k=1} A_{i,j,k}(t)\sin\frac{ix}{a}\sin\frac{jy}{b}\sin\frac{kz}{c}$.

Step 3. Expression for V is substituted into equation.

Step 4. Start the cycle by i,j,k from 1 to N.

Step 5. Taking the inner product of equation by the corresponding eigenfunctions $\varphi_i(x),\ \psi_j(y), \chi_k(z)$.

Step 6. Checking if λ belongs to the spectrum of the Laplace operator.

If sixth step is true:

Step 7. Verification of condition $\lambda = \lambda_1$.

If seven step is true:

Step 8. Solving of an algebraic equation with respect to $A_{i,j,k}(t)$.

If seven step is false:

Step 9. Initial data v_0, v_1 are multiplied by the eigenfunctions $\varphi_i(x),\ \psi_j(y), \chi_k(z)$.

Step 10. Solving of the ordinary differential equation of the second order, corresponding to the current number i,j,k in the cycle.

If the sixth step false:

Step 11. Initial data v_0, v_1, v_2, v_3 are scalar multiplied by the eigenfunctions $\varphi_i(x),\ \psi_j(y), \chi_k(z)$.

Step 12. Solving of the ordinary differential equation of the fourth order corresponding to the current number i,j,k.

Step 13. End of cycle by i,j,k.

Step 14. Founded Galerkin coefficients $A_{i,j,k}(t)$ are substituted into the approximate solution obtained in step 3.

Step 15. The resulting approximate solution is displayed as a graph of the solution with the animation over time from 0 to τ, with chosen fixed variable (for example z).

3. Numerical Experiment. Illustrate the described algorithm by several computational examples.

Example 1. Consider the problem

$$v(x,y,z,t) = 0, \quad (x,y,z,t) \in \partial\Omega \times \mathbb{R}, \tag{5}$$

$$\begin{aligned} &v(x,y,z,0) = \sin x \sin y \sin z, \quad v_t(x,y,z,0) = 10 \sin x \sin y \sin z, \\ &\frac{\partial^2 v}{\partial t^2}(x,y,z,0) = 3 \sin x \sin y \sin z, \quad \frac{\partial^3 v}{\partial t^3}(x,y,z,0) = \sin x \sin y \sin z, \end{aligned} \tag{6}$$

$$(\Delta - 2)\frac{\partial^4 v}{\partial t^4} + (\Delta - 1)\frac{\partial^2 v}{\partial t^2} + \frac{\partial^2 v}{\partial x_3^2} = 0. \tag{7}$$

It is required to find the numerical solution of problem (5) – (7) when $\lambda = 2, \lambda' = 1, \alpha = 1,$ in a domain $[0,\pi] \times [0,\pi] \times [0,\pi]$, $t \in [0,2].$

Eigenfunctions $\varphi_i(x),\ \psi_j(y), \chi_k(z)$ of the homogeneous Dirichlet problem for the Laplace operator in the domain $[0,\pi] \times [0,\pi] \times [0,\pi]$ have the form $\{\sin ix, \sin jy, \sin kz\}$. Obviously, in this case, equation (7) is not degenerate, therefore, the algorithm will take place in accordance with steps 11, 12 described in Section 2 of this article.

Graph of the solution is presented in picture 2 a.

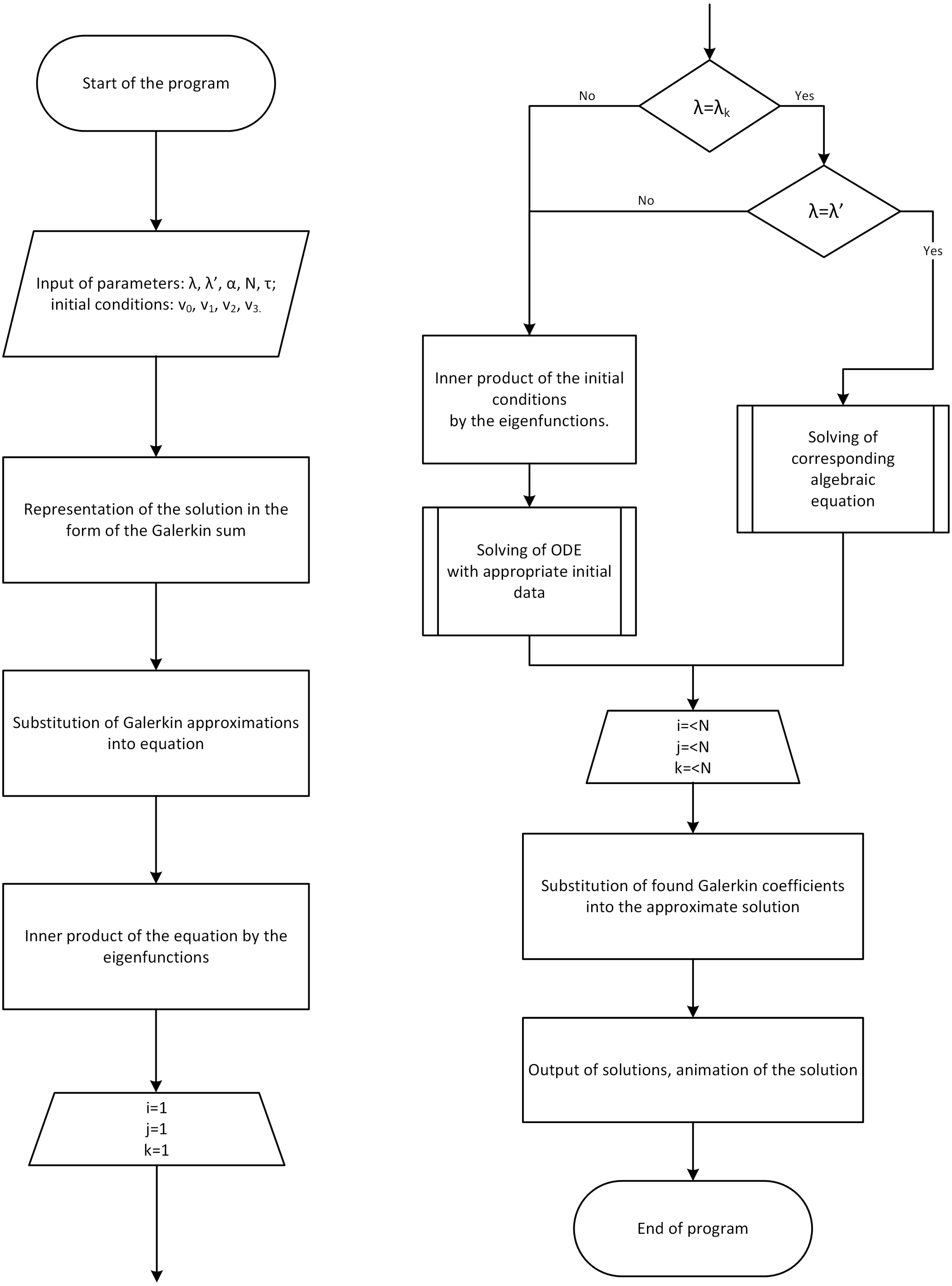


Fig. 1. A block diagram of algorithm

Example 2. Consider the problem

$$v(x;y;z;t) = 0; \quad (x;y;z;t) \in \partial\Omega \times \mathbb{R}; \tag{8}$$

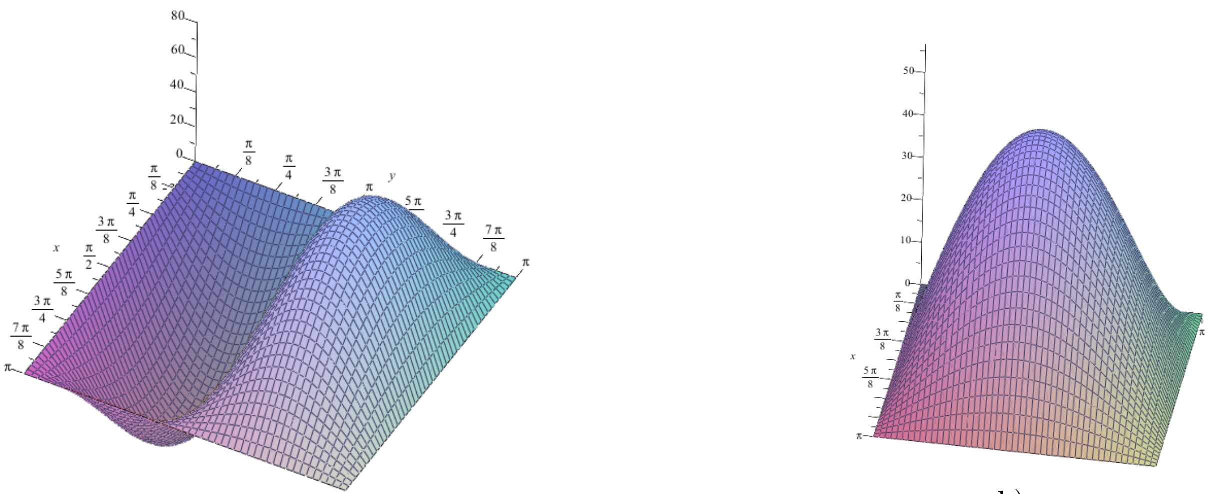
$$\begin{gathered} v(x;y;z;0) = 8\sin x \sin y \sin z; \quad v_t(x;y;z;0) = 0;1 \sin x \sin y \sin z; \\ \frac{\partial^2 v}{\partial t^2}(x;y;z;0) = 5\sin x \sin y \sin z; \quad \frac{\partial^3 v}{\partial t^3}(x;y;z;0) = \sin x \sin y \sin z; \end{gathered} \tag{9}$$

$$(\Delta + 4)\frac{\partial^4 v}{\partial t^4} + (\Delta + 4)\frac{\partial^2 v}{\partial t^2} - \frac{\partial^2 v}{\partial x_3^2} = 0: \tag{10}$$

It is required to find the numerical solution of problem (7) – (10) when $\lambda = 4; \lambda' = 4; \alpha = 1;$ in a domain $[0;\pi] \times [0;\pi] \times [0;\pi]$, $t \in [0;3]:$

Eigenfunctions $\varphi_i(x); \ _j(y); \chi_k(z)$ of the homogeneous Dirichlet problem for the Laplace operator in the domain $[0;\pi] \times [0;\pi] \times [0;\pi]$ have the form $\{\sin ix; \sin jy; \sin kz\}$. Obviously, in this case, equation (10) is degenerate, therefore, the algorithm will take place in accordance with step 8 described in Section 2 of this article.

Graph of the solution is presented in picture 2 b.



a) b)

Fig. 2. a) Solution from example 1; b) solution from example 2

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ВЫЧИСЛИТЕЛЬНЫЙ ЭКСПЕРИМЕНТ ДЛЯ ОДНОЙ МАТЕМАТИЧЕСКОЙ МОДЕЛИ ИОННО-ЗВУКОВЫХ ВОЛН

А.А. Замышляева, А.С. Муравьев

В статье рассмотрена математическая модель ионно-звуковых волн в плазме во внешнем магнитном поле. Данная математическая модель может быть редуцирована к задаче Коши для уравнения соболевского типа четвертого порядка с полиномиально $(A;p)$-ограниченным пучком операторов. Следовательно применимы абстрактные результаты по разрешимости задачи Коши для такого уравнения. В статье сформулирована теорема об однозначной разрешимости задачи Коши – Дирихле. На основе теоретических результатов был разработан алгоритм для численного решения задачи, основанный на модифицированном методе Галеркина. Алгоритм реализован в среде Maple. В конце приведены примеры, в которых решение получено при помощи разработанной программы.

Ключевые слова: математическая модель; ионно-звуковые волны; метод Галеркина.

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